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Special Issue **50 years of Fuzziness in Europe**




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In memoriam **Lotfi Zadeh** (1921-2017)

# Remembering an old friend, with a new Beginning.

The volume you are reading is a reprint of the commemorative issue of the **Archives for Philosophy and History of Soft Computing**, published in 2017 to honor the memory of Lotfi A. Zadeh and recollect the scattered history of Fuzzy Logic in Europe. These ideas, after more than 50 years, continue to be both very “challenging” and “controversial”. The contemporary presence of these two features for a long time is usually a sign that a crucial problem has been met.
The questions crossed cannot be easily answered by using the existing tools (be they, formal, technical or conceptual). There is a need for smart and innovative contributions coming from many different points of view.

It is, as well, an opportunity to announce that, starting 2019, the journal will be published under the imprints of Palermo University Press, with the new, shortened title of
**Archives for Soft Computing**.

**Rudolf Seising** and **Marco Elio Tabacchi**
Editors, *Archives for Soft Computing*

# Index

Editorial: **50(ish) years of Fuzzy Logic in Europe** **1**
**In memory of Lotfi Asker Zadeh**

**The introduction and development of fuzzy sets theory** **5**
**and applications in Finland**
Christer Carlsson, Vesa Niskanen, Hannu Nurmi

**Fuzziness in Italy - Traces of a scattered history** **15**
Gianpiero Cattaneo, Giulianella Coletti, Antonio Di Nola, Mario Fedrizzi, Giangiacomo Gerla, Gabriella Pasi, Marco Elio Tabacchi, Settimo Termini, Aldo Ventre

**The first steps in fuzzy set theory in France forty years ago (and before)** **47**
Didier Dubois, Henri Prade

**Fuzzy Sets and Fuzzy Logic in Austria** **61**
Erich Peter Klement

**Fuzzy systems research in Hungary – a subjective story** **77**
Laszlo T Koczy

**A Brief History of Fuzzy Logic in the Czech Republic** **103**
**and Significance of P. Hájek for Its Development**
Vilém Novák

**A Concise Glance at the History of Fuzzy Logic in Czechia** **119**
**(with a glimpse of the origin of data- mining - the GUHA method)**
Vilém Novàk

**The Gradual Arrival of Fuzziness in Switzerland** **135**
Edy Portmann

**On how Fuzzy Logic began in Spain (1974-1980)** **145**
Enric Trillas

**50 Years of Fuzzy Sets in Germany** **155**
Hans-Jürgen Zimmermann

# 50(ish) years of Fuzzy Logic in Europe In memory of Lotfi Asker Zadeh

Editorial

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## 50 and counting in Europe

The original idea of this special issue was to remember the important date of the 50 years elapsed from the publication of the 1965 Lotfi's seminal paper. Some occasional technical problems delayed the publication, but we must confess that these problems could have been solved, since almost all the papers were ready; what perhaps did not push the editors from trying to respect the date on the spot was a certain attitude to play with the idea that also the deadline could be respected fuzzily. And, anyway, we wish to recall once more that Lotfi used the term for the first time - in published form - in the beautiful 1962 paper which, for the acuteness of many of his epistemological comments, is still valuable today not only for historical reasons.

> «There are some who feel that this gap [the one between animate and inanimate Systems] reflects the fundamental inadequacy of the conventional mathematics – the mathematics of precisely-defined points, functions, sets, probability measures, etc. – for coping with the analysis of biological systems, and that to deal effectively with such systems, which are generally orders of magnitude more complex than manmade systems, we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions. Indeed,



Figure 1: Lotfi Zadeh in the late 1950s as instructor at Columbia University in New York (photo: Lotfi Zadeh Archive by Rudolf Seising).

> the need for such mathematics is becoming increasingly apparent even in the realm of inanimate systems, for in most practical cases the a priori data as well as the criteria by which the performance of a man-made system is judged are far from being precisely specified or having accurately-known probability distributions.» [1]

So, paradoxically, to respect "fuzzily" the 50th year of Information Control Paper, would correspond to respecting crisply the 55th year of the first appearance of the term *fuzzy*. We thought that all these unusual circumstances, coincidences (and scattered comments) could have enjoyed Lotfi. Now we are profoundly sad that our delay has impeded him to see what we had prepared in his (and his ideas) honour. We cannot, then, but present these pages also in his memory.

And, for doing this, let us add something. As the same Lotfi has stressed in many occasions, the crucial point of his innovative ideas is to consider that the change of perspective would at the same time force us to think anew to many questions - both (apparently) settled and (completely) unsettled.

[1] From Circuit Theory to System Theory, Proceedings of the IRE, May 1962, 50, p.857

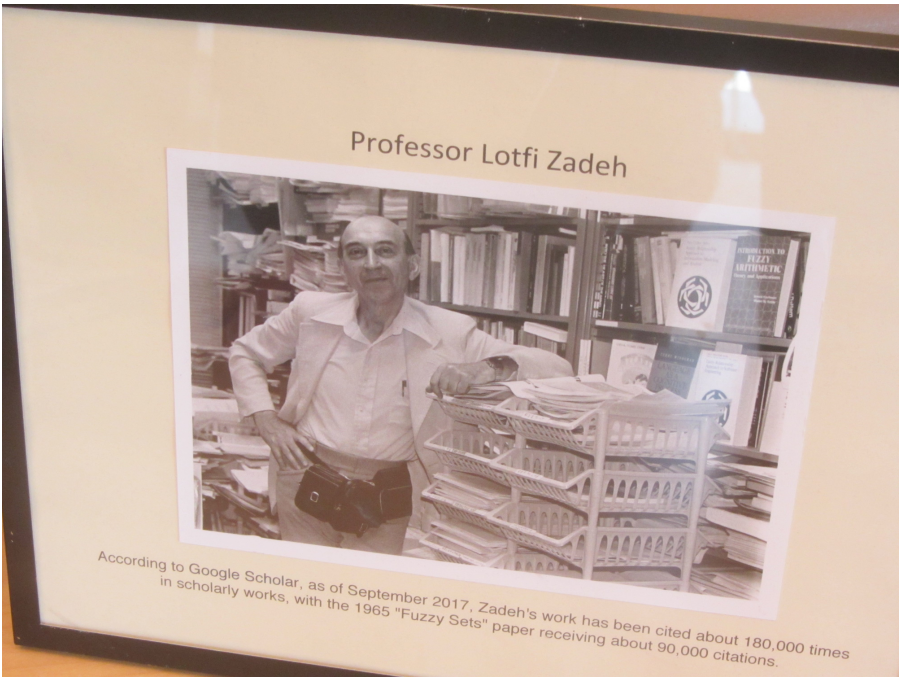


Figure 2: Lotfi Zadeh in his office at UC Berkeley, Evans Hall, in the late 1980s (photo taken at the Commemoration ceremony for Lotfi A. Zadeh, Wozniak Hall, EECS Dept. UC Berkeley, September 8, 2017; Lotfi Zadeh Archive by Rudolf Seising).

> «I would like to stress that there is much more to Fuzzy Logic than multi-valuedness of truth.»

he answered to a question in a 1995 interview and, to another question regarding why he was «down to earth» in his own approach to solving problems, his reply was:

> «I've been conscious for a very long time that the real world is very complex. There's always a tendency to oversimplify.»[2]

So, by remembering him, in this moment of sadness, what we want to stress is not only the import of his specific contributions but his crucial, general, suggestions. The world is very complex. We have the tendency to simplify. In some cases the simplification helps, in many other cases simplification impedes

[2] *http : //www.azer.com/aiweb/categories/magazine/24_folder/24_articles/24_fuzzylogic.ht*



Figure 3: Lotfi Zadeh in his office at UC Berkeley, Soda Hall, August 2011 (photo by Rudolf Seising).

to see things as they are. Approximation is an essential aspect of the way in which we approach and know the world, a very complex world. Substantially, this is the essence of fuzziness. By taking it into account we can understand better many things and, simplifying our models, extend the range of what we can understand. We desire to afford an extreme tribute to Lotfi Asker Zadeh, by stressing the importance of these methodological lessons.

# The introduction and development of fuzzy sets theory and applications in Finland

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## The introduction of fuzziness in Finland

The introduction and development of fuzzy sets theory and applications in Finland happened by pure chance at an unlikely moment in time. There was an exchange agreement in place between Finland and Romania in 1974 to accept researchers for 2 week visits to get to know useful people for scientific cooperation in both countries. The program paid for all travel cost and the host country for hotel and per diem; thus as Christer Carlsson was offered an invitation to go as an exchange researcher to Bucharest he – of course – accepted at once. At that time he was listed by Academy of Finland as a cybernetician in their exchange programs with socialist Eastern Europe (in fact, his doctoral thesis was in Operational Research, Systems Theory and Management Science but this was (loosely/vaguely) accepted as cybernetics).

In Bucharest the first morning was spent at a very high level meeting (on the level of the Academy of Science) and it turned out that the political people could not communicate with him as nobody was fluent in English and he did not know any French (the unofficial second language among educated people in Romania). Thus the man at the end of the table gave orders to find somebody who knew English and after a while Dr Constantin Negoita appeared and told him that the high level guys did not know what to do with him and that a move over to the Academy of Economic Studies to find some nice optimization work to do was in order. At his laboratory there was a bohemian guy who was a talented cello player and an extreme mathematician, Dan Ralescu. His first words were – "good, you got saved from the politicians (could be interpreted as "the people who do not know anything") – do you know anything about fuzzy sets?" Thus two weeks were spent working through the theory and some applications of fuzzy sets; Christer Carlsson's contribution was to share insights about real world, industrial planning, problem solving and decision making; time was also spent discussing and arguing about the few benefits and multiple shortcomings of the socialist economic system.

The next year – 1975 – there was an international conference on general systems theory in Bucharest, Dr C V Negoita was on the organizing committee and made sure that Christer Carlsson participated in the conference. In Bucharest Hannu Nurmi, who had defended his thesis in Political Science in 1974, attended the same conference (in typical Finnish fashion it turned out that Carlsson and Nurmi lived almost as neighbours and travelled on the same bus to work every morning but had to go to another country in order to get to know each other). One of the results of

this conference was that it was decided in the fall 1975 to start a study group on fuzzy sets in Turku. The driving force was Professor Olavi Hellman at University of Turku, he was professor of applied mathematics and a keen developer of operational research applications for industry and the Finnish defence forces. He had heard a few things about the theory of fuzzy sets and thought that the mathematics was sloppily done and could be improved. Thus work got started on this by first translating and reading up on the text book by Negoita and Ralescu [*Multimi vagi si aplicatiile lor* – re-published as *Applications of Fuzzy Sets to Systems Analysis* by Birkhäuser Verlag in 1975] – a bit of a challenge as the book was in Romanian but as Olavi Hellman noted, "mathematics is a universal language". Again we had a bit of luck, there was a German language lecturer at Åbo Akademi university who had studied Romanian as the minor for her Master's degree in Linguistics and she translated the Romanian text to German as an exercise; this we could read and in this way we got a first comprehensive text on the theory of fuzzy sets. Our first step was to decide on a Finnish concept for the theory of fuzzy sets and we decided on "sumean joukon teoria" (there were variations over the years, the version "sumea joukko teoria" was interpreted as the theory being fuzzy ("sumea") and was not even proper Finnish).

Later it turned out that our efforts were not the first presentations of fuzzy sets in Finland – prof Hans Blomberg at the Systems Analysis Laboratory at the University of Technology in Helsinki had invited Lotfi Zadeh to give a talk in 1970; the ideas of fuzzy sets were not well received – one of the participants described the theory as "poorly done probability theory". About this we did not know anything and as Prof Hellman was a renowned mathematician – and very sharp in his answers and comments if somebody wanted to question him – we were able to get going with building the foundations for Finnish applications of fuzzy sets theory.

The first official results were the Master's thesis by Jorma Mattila in applied mathematics in 1978 and the doctoral thesis by Christer Carlsson in 1977, which was one of the first proposals that fuzzy sets theory would be useful for management applications – this was later followed up with a chapter in the book that Hans-Jürgen Zimmermann, Brian Gaines and Lotfi Zadeh edited for the TIMS Studies in Management Science (1984).

Hans-Jürgen Zimmermann had started the EURO Working Group on Fuzzy Sets in 1974 as part of the EURO Association, the European organisation for Operational Research and he very quickly attracted a growing group of researchers to work on the new theory. At the same time he started both the European Journal of Operational Research and the Fuzzy Sets and Systems journal, both with North-Holland (now Elsevier) and both

have become very significant landmarks in the development of European research in both Operational Research and fuzzy sets theory. The EURO Working Group on Fuzzy Sets met at both the annual EURO conferences and at the biannual European Meeting on Cybernetics and Systems Research in Vienna; the latter meeting because Eastern European researchers could travel to Vienna but had visa problems to many EURO member countries.

Hans Zimmermann suddenly suggested at the EMCSR 1978 in Vienna that Christer Carlsson take over the chairmanship of the EURO WG on Fuzzy Sets; this was rather an overwhelming proposal for a very junior researcher but in true Finnish fashion he of course accepted the challenge. Carlsson was running the EURO WG for about six years after which Prof Marc Roubens (at Mons Polytechnique at that time) took over the chairmanship. "I cannot, of course, claim to have made landmark contributions to the theory of fuzzy sets but the network of researchers was started that first met and started cooperation and later became life-time friends and built significant contributions to both the theory and to the IFSA", Carlsson notes.

Here there is not space to go through the whole list, but a number of people that contributed to the development of fuzzy sets in Finland will be mentioned.

Prof. Jorma Mattila wrote his Licentiate thesis in applied mathematics (*Key Constructions for a Fuzzy Logic*) in 1984 and his doctoral thesis (*Calculus of Modifier Operators in Fuzzy Logic*) in 1989; the Finnish pioneer group on fuzzy sets acted as mentors for Mattila's work on fuzzy logic.

Adjunct Prof. Vesa A. Niskanen started his work on fuzzy systems in the late 1970's when he wrote his Licentiate's thesis on fuzzy logic in the Department of Philosophy at the University of Helsinki in 1980. He had heard about fuzzy systems from Professor Mauri Åhlberg in private discussions in the Department of Education in Helsinki and these discussions inspired him to study fuzzy systems. Vesa Niskanen defended his doctoral thesis (*A Fuzzy System with Linguistic Values: Some Logical and Methodological Considerations and Applications*) in 1986; in this context he met Professors Christer Carlsson and Hannu Nurmi for the first time; they were the recognized experts on fuzzy systems in Finland and were invited to serve as reviewers of Niskanen's thesis; Professor Hannu Nurmi acted as the ex officio opponent at Niskanen's dissertation.

Esko Turunen defended his doctoral thesis in applied mathematics – *A Mathematical Study of Fuzzy Logic. An Algebraic Approach* – in 1994 at the Lappeenranta University of Technology. With some simplification it can be said that the early contributions to fuzzy sets in Finland originated in

applied mathematics, operational research and logic.

Jari Kortelainen defended his doctoral thesis on fuzzy mathematics at the Lappeenranta University in 2000, and his thesis received in that year the distinguished Rolf Nevanlinna Prize. Other notable and active scholars to be mentioned are Tapio Rauma, Esko Juuso, Raija Koivisto, Kirsi Virrantaus, Heikki Koivo, Timo Honkela, Erkki Oja, Jarkko Niittymäki, Osmo Kaleva and Tero Joronen, inter alia. Both Antti Niemi and Heikki Puolakka wrote textbooks on fuzzy systems which were published by the National Board of Schools.

In general, it should be noted that the universities in Helsinki, Turku, Tampere, Lappeenranta, Jyväskylä and Oulu have been active in research on fuzzy sets and applications in a wide range of research areas.

Hannu Nurmi's first encounters with fuzzy systems took place in mid-1970's. The conceptual apparatus of fuzzy sets seemed appropriate for dealing with imprecise notions that abound in political and social sciences. The applications of fuzzy systems that Nurmi initially found most worth exploring were decision analysis and game theory (Nurmi 1975, 1977). It soon turned out, however, that before much progress could be made in these areas, some work on preference modelling was necessary. This, in turn, led to the field where Nurmi has done the bulk of his work on applied fuzzy sets, viz. collective decision making (1981a, 1981b). Most of this research has been conducted in close collaboration with Mario Fedrizzi (Trento), Janusz Kacprzyk (Warsaw) and Slawomir Zadrozny (Warsaw). Settimo Termini (Naples and Palermo) and Aldo De Luca (Naples) also were very important contacts in Nurmi's early work. Since mid-1980's Nurmi has visited the Systems Research Institute of the Polish Academy of Sciences regularly in order to pursue joint projects with Kacrpzyk and Zadrozny. This research collaboration is still ongoing.

One of the effective instruments to make the work on fuzzy sets known and accepted among Finnish researchers were a series of seminars called *Fuzziness in Finland* that were organized by Patrik Eklund and Jorma Mattila and got started in 1993. The seminars were organized on one of the (rather luxury) ferries on the route between Turku – the Åland Islands – Stockholm that the participants could enjoy travelling to and from the Åland Islands, where the seminar was run in beautiful surroundings. Vesa Niskanen joined the organizing team in 1994 when this cruising started from Helsinki. It was organized a last time from Helsinki to Tallinn in 2004 when Lotfi Zadeh was in Finland and participated in the seminar. This seminar attracted participants from the EURO WG network and from the contacts that the Finnish fuzzy sets researchers had built at conferences and

workshops. Early participants included Robert Fullér from the Eötvös Lorand University in Budapest, Mario Fedrizzi from University of Trento and Janusz Kacprzyk from the Systems Research Institute of the Polish Academy of Science. They all have co-authored numerous papers over the years with the Finnish participants - Patrik Eklund, Jorma Mattila, Hannu Nurmi, Pekka Isomursu, Osmo Kaleva, Jari Kortelainen, Harri Koskinen, Vesa Niskanen and Veikko Pohjola. Such visitors as Laszlo Koczy from the University of Technology in Budapest, Burhan Turksen from University of Toronto, Junzo Watada from Waseda University, Toshio Fukuda from Nagoya University and Kaoru Hirota from Tokyo Institute of Technology participated in the *Fuzziness in Finland* seminar in the later years.

Robert Fullér has spent a total of more than 6 years in Finland as a Visting Researcher, Visiting Professor and then as a Finnish Distinguished Professor 2008-2010 with funding from Tekes, the Finnish Funding Agency for Innovation. His research platform in Finland has been the Institute for Advanced Management Systems Research at Åbo Akademi University that Christer Carlsson started in 1992. Robert Fullér, now a Professor at the Obuda University in Budapest, has published about 120 papers with Finnish co-authors, many of them widely and frequently cited, and has been instrumental in making Finnish work on fuzzy sets known to an international audience. Some of the key contributions were collected in text books published by Springer and Kluwer (Carlsson and Fullér (2002), Carlsson, Fedrizzi and Fullér (2003) and Carlsson and Fullér (2011)). Robert Fullér worked with IAMSR researchers in a number of research projects co-funded by the Finnish industry and Tekes, which gave a number of break-through results. One of the most important results was the findings – when working out better methods to plan very large investments (called giga-investments) – that the theory of real options would benefit from replacing the classical assumption on stochastic long-term variations (using Geometric Brownian Motion to represent future variations of up to 15 years) with possibility distributions on fuzzy numbers for the key parameters in the Black-Scholes formula for real options. The first results were published in *Fuzzy Sets and Systems* in 2003 and got accepted (first reluctantly) by the international Real Options Group as an alternative approach to handle long-term variations when working out the profitability of large, strategic investments. This inspired a series of doctoral theses at IAMSR that developed and improved the fuzzy real options methods: Peter Majlender (2004, one of Robert Fullér's students from Budapest), Mikael Collan (2004, now a Professor at Lappeenranta University of Technology), Francisco Alcaraz (2006, now at National Bank of Ireland), Markku Heikkilä (2009, now

postdoc at IAMSR), Jozsef Mezei (2011, one of Robert Fullér's students from Budapest) and Xiaolu Wang (2015, now postdoc at IAMSR). This list shows in a nutshell how innovations introduced by fuzzy sets can change the applicability of classical theory and open up new roads for research.

The Finnish fuzzy sets group played a key role in the International Fuzzy Systems Association to secure its global operations. The banking rules had changed in the EU which made it problematic to transfer payments of membership fees and the proceeds from international conferences to IFSA that in legal terms was an informal association. Vesa Niskanen was the Secretary of IFSA 1999-2003 and took measures to change the status of IFSA to become a registered association according to Finnish law. This changed was approved by the IFSA Council and the economic infrastructure of IFSA was secured by the Åbo Akademi University that accepted to serve as the economic administrator of IFSA on a *pro bono* basis. As a result of this change IFSA has now been able to collect membership fees, the surplus from the international conferences and donations and to pay grants and awards to young researchers of the member societies of IFSA. Niskanen also organized Lotfi Zadeh's nomination to act as the Honorary President of IFSA as well as collected statistical data on IFSA members for the IFSA 2005 Congress in Beijing as Chair of the IFSA Information Committee.

Vesa Niskanen registered the IFSA Society *North European Society for Adaptive and Intelligent Systems* (NSAIS) in Finland by the request of some European scholars. Kauko Leiviskä was the first President and Niskanen is still acting as its Secretary; Pasi Luukka is now the President. NSAIS was sponsored by Omron Finland for many years because Niskanen was cooperating with this company.

Christer Carlsson, one of the *Founding Fathers of IFSA* in 1985, was elected IFSA Vice President in 2009, President-Elect in 2011 and served as President of IFSA 2013-15.

Niskanen met Prof. Jorma Mattila from Lappeenranta University in the late 80's in various seminars, and these meetings initiated their future cooperation and friendship in many ways. In those days Niskanen also started cooperation with Prof. Veikko Pohjola at University of Oulu and Prof. Raija Koivisto at VTT in Tampere.

In the early 90's Niskanen met Director Eero Silvennoinen from Tekes, the Finnish Funding Agency for Innovation, in the Finnish Artificial Intelligence conference in Otaniemi, and after their discussion Director Silvennoinen was so interested in fuzzy systems that he initiated in Tekes a technology programme on artificial intelligence. This programme, which received substantial funding from Tekes, was known as Adaptive and

Intelligent Systems Technology Programme, and from the beginning such scholars in fuzzy systems as Prof. Carlsson and Prof. Patrik Eklund (then and now at University of Umeå in Sweden) were active persons in this venture. This programme was so successful that it was continued a few years more than it was first planned in the 90's.

Niskanen had many appearances in the Finnish mass media on fuzzy systems since the 90's, and Lotfi Zadeh also attended these interviews several times. The Finnish broadcasting company (YLE) even produced a special TV program (two times 45 minutes) on fuzzy systems. In this manner a broader Finnish audience got familiar with fuzzy systems and Prof. Zadeh.

Niskanen organized three summer schools in Helsinki on fuzzy systems in 2000-2001 and 2004 with such distinguished keynote speakers as Lotfi Zadeh, Teuvo Kohonen, Jaakko Hintikka, Laszlo Koczy and Christer Carlsson.

Since the 90's, Niskanen has also acted as an ex officio opponent or reviewer to several Master's and Doctoral' theses on fuzzy systems in Finland. In this context Niskanen started cooperation with Rector Matti Pursula and Dr. Jarkko Niittymäki in the Helsinki University of Technology and later with Prof. Kirsi Virrantaus. Niskanen also organized that Lotfi Zadeh was once ex officio opponent at the University of Tampere.

Niskanen paid his first visit to BISC Group in Berkeley in 1995. In 1997 Tekes invited Prof. Zadeh to visit a conference organized by the foregoing technology programme, and on this occasion started the closer cooperation, and later the family-level friendship, between him and Niskanen. Since that year, Niskanen has visited many times in Berkeley, and he has also hosted with Prof. Carlsson and Tekes Zadeh's many visits to Finland. Through Niskanen's and Lotfi Zadeh's initiatives Tekes was able to negotiated an agreement with UC Berkeley and has partly funded the BISC Group.

Niskanen's other central international contacts, which stem from the late 80's, include Professors Laszlo Koczy (Hungary), Robert Fuller (Hungary), Junzo Watada (Japan), Kaoru Hirota (Japan), Toshio Fukuda (Japan), Burhan Turksen (Canada & Turkey), Janusz Kacprzyk (Poland), Witold Pedrycz (Canada), Zenn Bien (Korea, also cooperation with KOSEF), Paul Wang (USA), Masoud Nikravesh (USA), Patrik Eklund (Finland & Sweden,), Mustafa Gunes (Turkey), Jon Teichrow (USA) and Rudolph Seising (Spain & Germany).

In the beginning of this millennium Niskanen initiated Prof. Zadeh's membership as a foreign member of the Finnish Academy of Sciences and

Letters and with the important contribution of Prof. Jorma Mattila that Prof. Zadeh was appointed Doctor Honoris Causa at the University of Lappeenranta.

In 2005 Lotfi Zadeh, Niskanen and Masoud Nikravesh made plans for organizing the 40th anniversary of fuzzy systems in Berkeley. Niskanen and Nikravesh then selected members to organization committee and made the list of invited guests as well as organized other practical issues with some of their colleagues. These celebrations took place in Berkeley in autumn 2005.

Niskanen's later activities include memberships in the Learning Business Plan Project with Prof. Paula Kyrö in Aalto University (funded by Ministry of Trade and Industry), Kuljen Project with Prof. Raija Koivisto in VTT (funded by Tekes) and Project on Fuzzy Cognitive maps (with Laszlo Koczy and Miklos Hartwagner and funded by Ecole Centrale in Paris). In addition to scientific articles, he has also written books on fuzzy systems, one in Finnish published by WSOY and the other in English published by Springer.

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# Fuzziness in Italy – Traces of a scattered history

Gianpiero Cattaneo[1], Giulianella Coletti[2], Antonio Di Nola[3], Mario Fedrizzi[4], Giangiacomo Gerla[3], Gabriella Pasi[1], Marco Elio Tabacchi*[5,6], Settimo Termini[5], Aldo Ventre[7]

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**Abstract** The history of Fuzziness in Italy is varied and scattered among a number of research groups. As a matter of fact, "fuzziness" spread in Italy through a sort of spontaneous diffusion, and, also subsequently, no one felt the need to create some "national" common structure like an Association or similar things. Since a cohesive retelling would be next to impossible, a few members of the Italian fuzzy community have been asked to recount their experience and express their hopes for the future.

## 1 Introduction

The history of Fuzziness in Italy is varied and scattered among a number of research groups. As a matter of fact, "fuzziness" spread in Italy through a sort of spontaneous diffusion, and, also subsequently, no one felt the need to found some "national" common structure like an Association or similar things. The European Association - at its level - played, later, a brilliant role. Since a cohesive retelling would be next to impossible, a few members of the Italian fuzzy community have been asked to recount their experience and express their hopes for the future. Although partial and, surely, incomplete, what follows can provide an idea of the richness and variety of the paths followed. A richness due perhaps to just the chaotic and unplanned way of its diffusion.

Since not even a chronological order (let alone a semantic or prioritising one) can be attempted, the contributions are grouped following loose thematic connections mixed with author's lexicographic order.

## 2 Gianpiero Cattaneo: Brower–Zadeh structures according to a personal approach

This small note will be an information document about my interest developed in several years of research about fuzziness, with induced results on roughness. All started around the seventies of the last century that I spent as an associate professor at the University of Calabria (south Italy), where a colleague from the Department of Sociology introduced me to the world of fuzziness by the knowledge of two, for me very important, papers about this argument: the seminal Zadeh's paper [1] and a De Luca–Termini contribution [2]. Before this, I was involved in theoretical researches about foundations of quantum mechanics according to the so-called unsharp approach, especially from the quantum logical point of view, but I immediately realized a deep formal analogy between the mathematical structures involved in these two approaches where the classical dichotomy crisp–fuzzy corresponds to the quantum dichotomy sharp–unsharp. It is only a terminological difference in describing the relationship between certain and uncertain, precise and unprecise, sharp and vague in two different contexts.

Formally, I was interested to the abstract approach to these arguments based on the algebraic structure of poset, which is a lattice in the fuzzy set theory and a partial ordered structure which is not a lattice in the quantum case. But, in studying the above mentioned contributions in fuzzy set theory I was impressed by the fact that in the Zadeh paper [1] the (distributive, complete and bounded) lattice $\Sigma, \wedge, \vee, 0, 1$ is equipped with a Kleene orthocomplementation mapping $' : \Sigma \to \Sigma$, which generalizes the orthocomplementation of standard set theory (for arbitrary $a, b \in \Sigma$, (oc-1) $a = a''$ strong double negation law; (oc-2) $a \leq b$ implies $b' \leq a'$ contraposition law; (K) $a \wedge a' \leq b' \vee b$

Kleene condition). Differently, into the De Luca and Termini paper [2] the lattice $\Sigma$ can be equipped with a Brouwer (intuitionistic) orthocomplementation mapping $^\sim : \Sigma \to \Sigma$, which is a different generalization of standard orthocomplementation (for arbitrary $a, b \in \Sigma$ (woc-1) $a \leq a^{\sim\sim}$ weak double negation law; (woc-2) $a \leq b$ implies $b^\sim \leq a^\sim$ contraposition law; (woc-3) $a \wedge a^\sim = 0$ non contradiction law).

I was soon intrigued by the investigation of a poset structure $\Sigma, \leq, ', ^\sim, 0, 1$ in which both these orthocomplementations are involved, with a further interconnection rule about these two, formalized as the law: (ir) for every $a \in \Sigma$, $a^{\sim\prime} = a^{\sim\sim}$, which is satisfied both in standard fuzzy set theory and in unsharp quantum mechanics. The poset requirement, instead of the lattice one, is due to the fact that in the quantum case the partial order does not induce the lattice operations. Notice that in the lattice case, where the lattice meet $\wedge$ and join $\vee$ are algebraic realizations of the logical connectives And and Or, the orthocomplementations $'$ and $^\sim$ are also interpreted as two kinds of non standard negations in the algebraic semantic of some propositional logic, in [3] interpreted as Not and Falsehood (sometimes this latter also as Impossibility).

The first approach to this kind of structure in discussing arguments related to fuzziness can be found in [4], but during my staying in Calabria I started a collaboration with two researchers/collaborators of this university which produced several papers about this poset (resp., lattice) structure, from me defined as Brouwer-Zadeh (BZ) poset (resp., lattice). The first in time was the paper [5], presented in the first Napoli meeting on Mathematics of Fuzzy Systems (1984), where taking inspiration from the concrete case of the lattice–algebraic structure of fuzzy sets on a reference space ($\Sigma = [0,1]^X$, with $X$ the reference space), the general BZ poset structure is introduced, with the analysis of all the formal induced machinery: the definition of crisp (exact, sharp) elements as those elements that coincide with their double Brouwer negation ($f \in \Sigma$ is crisp iff $f = f^{\sim\sim}$), where the two nonstandard orthocomplementation coalesce in a unique standard one ($\forall f$ crisp $f' = f^\sim$), satisfying besides the strong double negation and the contraposition laws, also the non contradiction and the excluded middle laws.

On the basis of this structure, the two modal operators obtained by suitable compositions of orthocomplementations, necessity $\nu(a) = a'^\sim$ and possibility $\mu(a) = a^{\sim\prime}$, which are both crisp elements, are introduced showing that they satisfy the semantical algebraic properties required by modality, but in the context of a non Boolean lattice, such that both coalesce to the identity on crisp elements ($\forall f$ crisp $\nu(f) = \mu(f) = f$). Moreover, the collection of all necessity–impossibility pairs $(\nu(a), \mu(a)') = (\nu(a), \nu(a'))$ gives rise to a three–valued Lukasiewicz poset, with corresponding BZ structure since also in this case it is possible to introduce a Kleene negation $-(\nu(a), \nu(a')) = (\nu(a'), \nu(a))$ and a Brouwer negation $\sim (\nu(a), \nu(a')) = (\nu(a'), \mu(a))$ (as consequence of the

modal required relation between possibility and necessity $\mu(a) = \nu(a')'$). If the original BZ structure $\Sigma$ is a lattice, then also the induced three–valued Lukasiewicz structure is a lattice.

In the same meeting also the first contribution to concrete BZ poset structure in the unsharp quantum mechanical situation has been presented in [6], where the role of fuzzy elements are played by the so-called effect operators on a complex separable Hilbert space, which as said before has a structure of BZ poset which is not a lattice.

All the BZ definitions, in the poset and not lattice context, are introduced with the investigation of the corresponding properties. In particular, it is shown that the collection of sharp elements coincide with the set of all orthogonal projections, which is an orthomodular lattice despite the original structure is a poset. This leads to the fact that also the induced three-valued Lukasiewicz BZ structure is a lattice.

As widely discussed in some papers of physical content [7,8], the BZ poset of effect operators can be related to the Ludwig approach to unsharp quantum mechanics, whereas the induced three-valued Lukasiewicz BZ lattice is the natural environment of the Piron approach to the same argument.

After an abstract study about non usual orthocomplementations in the fuzziness context [9], the main paper about a treatment of BZ poset structures has been done in [10], and for the specific quantum case see also [11].

In 1991 I started a collaboration with the group of Logics from Florence, directed by Marisa Dalla Chiara and with some of her collaborators, Roberto Giuntini and Federico Laudisa, about BZ structures applied to the logics of unsharp quantum mechanics. I mention here the first contributions to this argument [12–14], with interesting application to physics [15], and a survey which can be found in [16]. In the same period I moved to the University of Milano (from 1984 to 1999) and then to the new University of Milano-Bicocca (from 1999 till my retirement occurred some years ago). In these two new contexts I created new research groups whose main interest was the connection of fuzzy set theory with rough set theory, always in the algebraic context of BZ structures.

As discussed above, a BZ structure, either of poset (unsharp quantum case) or of lattice (fuzzy set case), with the two non standard Kleene $a'$ and Brouwer $a^\sim$ orthocomplementations can be considered the right abstract environment to describe fuzziness. The induced two operators of $\nu(a) = a'^\sim$ and $\mu(a) = a^{\sim\prime}$ are not only the algebraic realizations of modal necessity and possibility, but have the interesting property of being the best lower and upper crisp approximations, respectively, of the approximable element $a$. This means that on the pair $(\Sigma, \Sigma_e)$ consisting of the BZ lattice $\Sigma$, now interpreted as the collection of the elements which can be approximated, and the ortholattice $\Sigma_e$ of corresponding crisp elements, considered as the exact ones, for any approximable element $a \in \Sigma$ the pair $r(a) = (\nu(a), \mu(a))$ is its rough approximation by exact

elements since $\nu(a)$ (resp., $\mu(a)$) is the best approximation from the bottom (resp., top) by crisp elements. Indeed, $\nu(a) \in \Sigma_e$ (resp., $\mu(a) \in \Sigma_e$) is a crisp element that furnishes an inner (resp., outer) approximation of $a$ since $\nu(a) \leq a$ (resp., $a \leq \mu(a)$) and which is the best one since if $f \in \Sigma_e$ (resp., $g \in \Sigma_e$) is another inner (resp., outer) crisp approximation of $a$, i.e., $f \leq a$ (resp., $a \leq g$), then $f \leq \nu(a)$ (resp., $\mu(a) \leq g$).

A first exposition of these new version of BZ structures can be found in [17], whereas the study of a generalization of this argument can be found in [18] (and see also [19]). Always linked to this argument of roughness in the abstract context I mention the following papers [20–22], with an interesting contribution about the discussion of the role of intuitionistic negation in the Atanassov approach to the so-called intuitionistic fuzzy sets [23]. I conclude with the last citation [24], in which instead of considering as original structure the one based on a Kleene and Brouwer orthocomplementations, the starting point is a structure, called of MDS5 algebra, based on a de Morgan orthocomplementation (i.e., negation) and a modal necessity operator satisfying axioms N, K, T, and 5 of modal logic (which does not coalesce to the identity since the negation is not the Boolean one). The consistency of this algebraic model is proved, since the usual fuzzy set theory can be equipped in a natural way with a MDS5 structure.

## 3 Antonio Di Nola: Fuzzy Logic as a Logic: the wisdom of fuzzy sets

The novelty of Zadeh' idea of *Fuzzy Set* lies not just in providing a theory for vague concepts, but in offering a representation of them via functions. Fuzzy logic was conceived by L. Zadeh in 1965, [1], as an attempt at dealing with vagueness, with a marked slant towards practical applications. Subsequent developments in that field were mostly driven by engineering practice, intuition, and persistence.

Just to remark the importance of the theory of fuzzy sets, we note that every time that one cannot give a sharp input to a system then a more general framework is needed for the system. This, sometime, can be done using fuzzy systems. Such a machinery helps to cover more cases than classical systems do. Thus the problem of a theoretical foundation of fuzzy sets and of fuzzy systems arises.

To approach any theoretical problem it is important to have open mind to use several different theoretical tools.

It is important to remember that in science "tout se tient". That is, same ideas can appear under many different dresses and many components play a proper role in the picture.

Representation calls for a formal treatment. Formalization is abstraction,

and abstraction, as output, gives several incarnations of the same concept. For example, look at Boolean Algebras. Boolean algebras are the algebraic models of Classical Logic and their role, in mathematics, is relevant in Ring Theory, Topology, Functional Analysis and Probability, among other fields of Mathematics. This means that the deep structure of classical logic is reflected into deep mathematical objects in many different ways. Any generalization of classical logic has to pay a price to the profound interconnection between its algebraic treatment, by comparing the structure of its Lindenbaum Algebra and other mathematical structures. The rich universe of symmetries of Boolean Algebras, in some extent is lost, but, in contrast, new links between Lindenbaum algebras of non classical logics and other field of Mathematics appear.

The Min-Max fuzzy logic, as a first proposed fuzzy logic, had a too restrictive point of view, as it is testified by the very simple lattice structure of fuzzy sets as functional model of such a logic.

A great effort was done to provide algebraic semantics for fuzzy logic, in view to help people, oriented at applications, to better understand the fuzzy logic, as a logic, and to capture an elegant way to mathematically express the wisdom of the idea of fuzzy set.

One may ask: Why fuzzy sets? A first answer could be: to represent a special kind of information. How to use fuzzy sets? By using the mathematics of fixed classes of fuzzy sets. That is to represent the semantics of a fuzzy logic by an algebra of functions. Thus, fuzzy sets codify the information expressed by a precise language, starting from fixed axioms of a logic. The set of axioms represents the basic knowledge.

Many-valued logics are the logical instrument to use when dealing with more than two truth values. In particular, in the large class of many-valued logics, developed so far, a leading role has played by Lukasiewicz Logic. This logic is based on the Lukasiewicz t-norm as interpretation of conjunction.

Besides finding applications to the treatment of uncertain information, the infinite-valued calculi based on Lukasiewicz logic are very interesting for their deep relationships with diverse mathematical areas.

Here, we aim to show that Lukasiewicz Logic and its extensions are generalizations of classical logic which represent a suitable compromise between generality and profoundness of their algebraic models as MV-algebras, for example.

At early stage of fuzzy sets' age soon some interesting ideas arose. We like to mention the idea due to De Luca and Termini ( [25]) of *entropy of a fuzzy set*, idea which, later on, was developed by Mundici, [26], in a theory of states of algebras of Lukasiewicz Logic, as formalization of the concept of averaging truth values. Belluce in the paper [27] linked fuzzy sets with Lukasiewicz Logic, and in the paper [28] were proposed non-standard fuzzy sets, showing that by representing vagueness via functions one can consider also infinitesimals as truth values of a vague proposition. Some years later, Hajek ( [29]) giving the basic

theory of Fuzzy Logic as a logic, via $t$-norms operators, included Lukasiewicz Logic among the basic fuzzy logics.

All the above theories, starting from the end of the eighties, were developed also by people at Naples University, where a numerous research group was very active. It is worth to mention, among others, the work of A. Lettieri, S. Sessa, and A. Ventre. A little bit later, and still in Naples, the important seminar: *Order in Algebra and Logic* was held along ten years. The seminar had, among its interests also problems concerning the foundation of theory of fuzzy sets. The seminar was supported by the generosity of the Istituto Italiano per gli Studi Filosofici and by the work of the steering committee: E. Casari, M. Curzio, A. Di Nola, A. Macintyre, D. Mundici, G. Tallini. Several editions were guested abroad, at Oxford University, University of Galway, Barcelona, Marseille and of course at Naples. The neapolitan research group in fuzzy logic was essentially involved in studying Lukasiewicz Logic and MV-algebras.

MV-algebras, a non-idempotent generalization of Boolean algebras, have an impressive array of deep connections with several fields of mathematics, including inter alia lattice-ordered groups, approximately finite-dimensional C*-algebras, and rational polyhedral geometry. The interplay between algebra, geometry and proof-theory in Lukasiewicz logic, and its applicability to the management of information that is both vague and uncertain, are characteristic features of this kind of logic.

Establishing connections between MV-algebras and other areas of mathematics was a mainstream of investigation, see [30], [31].

In [32], Mundici found a categorical equivalence, $\Gamma$, between MV-algebras and lattice ordered abelian groups with strong unit. This theorem is the origin of a considerable number of remarkable results in the theory of MV-algebras. In the same vein, a different categorical equivalence was established in [33], where it was shown that the category of lattice-ordered abelian groups is equivalent to the category of perfect MV-algebras.

The representation theory of MV-algebras has a rich history. C.C.Chang proved the first representation theorem, [34], [35] . In the context of many-valued logic, the algebras of [0, 1]-valued functions have a special role. These are the semisimple MV-algebras, i.e. algebras without infinitesimals. Their functional representation was pointed out by Chang and Belluce. In [36] was provided a functional representation for all MV-algebras: any MV-algebra can be seen as an MV-algebra of functions from a set to an ultrapower of [0,1].

### 3.1 Scaling propositions: beyond groups

At the beginning of the new millennium the neapolitan research group on fuzzy logic was embodied in a larger gruop at the University of Salerno. There, new and younger people like B. Gerla, G. Lenzi, C. Russo and L. Spada started to

cooperate still on studying MV-algebras theory and their links with geometry, topology and functional analysis.

A further research direction was to characterize the class of structures generated by $[0, 1]$ in the language of MV-algebras enriched with the real product. The analogue Mundici's theorem for these structures, called $PMV$-algebras, was obtained in [37].

It seems quite natural to introduce *modules* over PMV-algebras. The structure of MV-modules is defined and studied in [38]. The MV-modules over $[0, 1]$ are particularly interesting, since they are categorically equivalent with a class of Riesz spaces.

The deep connection between Riesz MV-algebras and Riesz spaces with strong unit allows one to obtain new logical information using geometric and analytic methods. An analysis of the different aspects of this logic, from one side, provides the limit of an intermediate logic, the Rational Lukasiewicz Logic, on the other side MV-Riesz Logic, see [39], can be used to develop a logic for compact Hausdorff spaces via their categorical equivalence with norm-complete Riesz MV-algebras. All these examples witness that Riesz MV-algebras are a malleable notion, in need of a systematic investigation which could lead to significant advancements in the framework of Lukasiewicz Logic.

### 3.2 The geometry of fuzzy logic

Several studies are devoted to the concepts of algebraic geometry over fields applied to the theory of MV-algebras. According to Mundici, [40], rational polyhedra are the genuine algebraic varieties of the formulas of Lukasiewicz Logic, in a precise sense: zerosets of McNaughton functions coincide with rational polyhedra, see [40]. On the other hand, there are reasons to be interested in other MV algebras, because every MV algebra can be viewed as the Lindenbaum algebra of some many-valued logic, and as such, it has logical relevance. This is why some people from the research group on Logic at University of Salerno generalized somewhat the theory of Mundici to MV-algebras as general as possible, using the main tools of universal algebraic geometry, as developed in [41]. In [42] are shown the first results in this very promising direction.

## 4 Giangiacomo Gerla: my interest toward fuzzy logic

From the Eighties of last Century I have addressed my interest toward fuzzy logic as a consequence of contacts with the research group composed by A. Di Nola, A. Lettieri, S. Sessa and others at the Federico II University in Naples. This interest was originated by a more general interest in the naïve science of the common man which is qualitative in nature, in contrast with modern science, quantitative in nature. Indeed, at the same time, I investigated the possi-

bility of defining mathematical models of the naïve idea of space in which one assumes as a primitive the notion of region or solid body while the points are defined by suitable sequences of nested regions (*point-free geometry*).[1] These two interests are related since, as in some recent papers I emphasizes, fuzzy logic appears to be a promising tool to face point-free geometry and this provided one refers to languages containing predicate symbols such as "small", "close", "contained" whose intended meaning is geometrical in nature.

More specifically, I embrace the graded syntax approach to fuzzy logic since, in my opinion, it fits well with the argumentations in our everyday lives. This means that I refer to the famous paper by J. A. Goguen [44] and with the successive formalization proposed by J. Pavelka (see also the significant contributions of V. Novak [45]). In accordance with Pavelka, Gerla's researches are based on the systematic use of the theory of closure operators in a complete lattice (see [46]). This means that if $F$ denotes the set of formulas of a first order language, then a model is identified with a fuzzy subset $m$ of $F$, i.e. the fuzzy subset of formulas which are true in this model. In accordance, a semantics is a class $M \subseteq [0,1]^F$ of fuzzy subsets of $F$. In several cases, $M$ is the class of truth-functional valuations of the formulas in $F$. Now, in the graded approach instead of a set of axioms one considers a fuzzy subset $\tau : F \to [0,1]$ of axioms. The intended meaning is that, for every formula $\alpha$, $\tau(\alpha)$ is the constraint $[\tau(\alpha), 1]$ on the truth value of $\alpha$, i.e. the information "the truth value of $\alpha$ is greater or equal to $\tau(\alpha)$". In accordance, one says that $m \in M$ is a *model of* $\tau$ provided that $m \supseteq \tau$ and the *logical consequence operator* $C : [0,1]^F \to [0,1]^F$ is defined by the equation

$$C(\tau)(\alpha) = inf\{m(\alpha) : m \in M, m \supseteq \tau\}.$$

$C(\tau)(\alpha)$ represents the best possible constraint on the truth value of $\alpha$ we can derive from $\tau$. Notice that in this formalism one refers to the fuzzy subset of logically true formulas $C(\emptyset)$ and not to a classical subset of logically true formulas as in the traditional syntax approach to fuzzy logic.

A second operator is the *deduction operator* $D : [0,1]^F \to [0,1]^F$. Namely, the notion of a proof $\pi$ of a formula $\alpha$ under the fuzzy subset $\tau$ of hypotheses is defined and a constraint $c(\tau,\pi) \in [0,1]$ is associated with $\pi$, given $\tau$. Since different proofs of $\alpha$ can give different constraints, $D(\tau)(\alpha)$ is obtained by fusing all the possible constraints,

$D(\tau)(\alpha) = sup\{c(\pi) : \pi$ is a proof of $\alpha$ under the fuzzy subset $\tau$ of hypotheses$\}$.

Both $C$ and $D$ are closure operators.

[1] Both naïve logic and naïve geometry find their place alongside 'naïve physics' (see [43]).

Now, given a fuzzy semantics, the basic question is to find a deduction apparatus whose associated deduction operator $D$ coincides with $C$ (completeness theorem). It is evident that this operator should be in some way 'computable'. As matter of fact, the meaning of the completeness theorem for a logic is that the set of theorems of a decidable theory is effectively enumerable. In accordance, I propose to extend to the fuzzy framework the classical notions of 'computable', 'decidable', 'effectively enumerable', 'enumeration operator'.

At this regard, my idea is that to do this, we have to take into account the strong difference between classical logic, based on the finite set $\{0, 1\}$, and fuzzy logic, based on to the continuum [0,1]. This difference entails that while classical logic is related with algorithms giving the output after a finite number of steps (recursive arithmetic), fuzzy logic must be related with endless effective processes of approximation (recursive analysis). Then, he proposes the following definitions (see for example [47]).[2]

Denote by $[0,1]_Q$ the set of rational numbers in [0,1] and let $S$ be a nonempty set. Then a fuzzy subset $s : S \to [0,1]$ of $S$ is *effectively enumerable* (*co-enumerable*) if there is a recursive map $h : S \times N \to [0,1]_Q$ such that

i) $h$ is order-preserving (order-reversing) with respect to the second variable,

ii) $s(x) = lim_{n \to \infty} h(x,n)$ for every $x \in S$.

A fuzzy subset $s$ is *decidable* provided that is at the same time effectively enumerable and effectively co-enumerable. Notice that in this case, $s(x)$ can be any computable real number. Now, the kind of effectiveness required to the deduction operator $D$ of a fuzzy logic requires that if $\tau$ is decidable, $D(\tau)$ has to be effectively enumerable. This leads to the following definition. Denote by $FFS(S)$ the class of finite fuzzy subsets of $S$ with rational values, put $SEQ_f = FFS(S) \times F$ and denote by $s_f \vdash \alpha$ an element of $SEQ_f$. Moreover, define the relation $\ll$ by assuming that $s \ll s'$ provided that $s(x) < s'(x)$ for every $x$ such that $s(x) \neq 0$. Then, $H : [0, 1]^S \to [0, 1]^S$ is an *enumeration fuzzy operator in S* provided that there is an effectively enumerable fuzzy subset $w : SEQ_f \to [0,1]$ of $SEQ_f$ with rational values such that

$$H(s)(x) = Sup\{w(s_f \vdash x) : s_f \ll s\}.$$

I observe that the deduction operators in the main fuzzy logics is an enumeration operator. This entails that if $\tau$ is decidable, then $D(\tau)$ is effectively enumerable as required. In particular $D(\varnothing)$ is a recursively enumerable fuzzy subset and, in my opinion, this shows that the graded approach to fuzzy logic is immune of the strong difficulties arising from Scarpellini's theorem [48]. Indeed, notice that the fact that $D(\varnothing)$ is effectively enumerable is compatible

[2] My researches on the closure operators and on fuzzy computability have been largely developed in collaboration with L. Biacino of the Federico II University. More recently, several papers where written in collaboration with Cristina Coppola and Tiziana Pacelli of Salerno University.

with the fact that $\alpha \in F : D(\emptyset) = 1\}$ is not.

## 5 Giulianella Coletti: A bridge between fuzzy set theory and coherent conditional probability: a step toward armonizing theories for managing uncertainty

I'm a member of a group of researchers in probability theory, which follows the ideas, the suggestions and intuitions of de Finetti. As is well-known de Finetti's approach to probability (and in particular to conditional probability) trough coherence is particularly suitable for managing uncertainty in intelligent systems and in approximate reasoning. During the nineties we actively attended various international conferences in soft computing, where the different approaches to reasoning under partial, imprecise and vague information were animatedly discussed.

Previously I had come in contact with fuzzy set theory during interesting discussions with Settimo Termini, which at the end of the eighties was in Perugia as Professor of Cybernetics and Information Theory. However, fuzzy sets and fuzzy logic were still to me essentially an intellectual curiosity, an enrichment of my knowledge, but certainly not a research field. Instead, during the meetings mentioned before, Romano Scozzafava and I realised that it was necessary to have a single well-founded setting for handling problems such as the following:

> *A box contains $n$ balls of various sizes $s_1, ..., s_m$, with $m \leq n$. Under the hypothesis that "most balls are large", what is the probability that a ball drawn at random is small*?

The possible strategies for this aim were two:

- Define conditional probability in a not Boolean ambit, following the line of the logic school (see for instance Mundici [32,49], Hajek, Tulipani [50]) and find which results of probability theory continue to hold in this context, that in any case was limited to Lukasiewicz t-norm and t-conorm, surely not the best for managing probability problems.

- Propose an interpretation of fuzzy sets and fuzzy events in terms of conditional events and conditional probability, and use all the results in probability theory to manage probability on fuzzy events.

The second idea seemed to be the most profitable and so for some years Romano Scozzafava and I, with the important contribute of some of our pupils (in particular Barbara Vantaggi) coped with the construction of a bridge between the two theories. For that it was necessary to develope a model of

generalised inference, based on the theory of coherent conditional probabilities (essentially introduced by de Finetti) and the relevant extension problems (see [51–60]). Such theory is based on the following milestones:

– A conditional event is any ordered pair of Boolean events $E|H$, with $H \neq \emptyset$, which can be both true or false and are different only in the role they play ($H$ acts as an hypothesis). It is in fact essential to regard also the conditioning event $H$ as a *variable*, i.e., the status of $H$ in $E|H$ is not just that of something representing a given fact, but that of an uncertain event (like $E$) for which the knowledge of its truth value is not required.

– The uncertainty can be caused by different circumstances, but it is reducible in fact to a lack of complete information.

– The theory of reference (coherent conditional probability) is able to handle both domains without particular Boolean structure and partial or imprecise assessments.

In this context we proposed our interpretation of a fuzzy subset: we refer to the state of information (at a given moment) of a real (or fictitious) person that will be denoted by "You". Let $Z$ be a (not necessarily numerical) variable, with range $C_Z$, denote by $z$ the event $\{Z = z\}$, by $\varphi$ any *property* related to $Z$ and finally by $E_\varphi$ the Boolean event {You claim that Z is $\varphi$}. Consider the conditional events $E_\varphi|z,\ z \in C_Z$ and a coherent probability assessment on them. By a fundamental result in coherent probability theory, it follows that You may assign to each of these conditional events a degree of belief $P(E_\varphi|z)$, without any syntactical restriction.

So, we can define a fuzzy subset $E^*_\varphi$ of $C_Z$ as a pair $\{E_\varphi\ ,\ \mu_{E_\varphi}\}$ with

$$\mu_{E_\varphi}(\cdot) = P(E_\varphi|\cdot).$$

For this interpretation the semantic of the membership degree of $z$ is very clear: $\mu_{E_\varphi}(z)$ is the probability that You claim that $Z$ is $\varphi$, in the case that $Z$ assumes the value $z$. Moreover, by using only results of coherent conditional probability theory, we could find many (unfortunately not all) concepts and the related results of the theory of fuzzy sets, such as the following: given two fuzzy subsets $E^*_\varphi,\ E^*_\psi$ related to the same variable $Z$, with the events $E_\varphi,\ E_\psi$ logically independent, the coherent values for $\mu_{\varphi\wedge\psi}(z) = P(E_\varphi \wedge E_\psi|z)$ and $\mu_{\varphi\vee\psi}(z) = P(E_\varphi \vee E_\psi|z)$, for any given $z$ in the range of $Z$, can be obtained by any Frank t-norm and their dual t-conorms.

Just for citing one of the mentioned limitations of this interpretation, I recall that: coherence is violated if any t-norm and t-conorm different from those in the Frank's class are used for computing $\mu_{\varphi\wedge\psi}(z)$ and $\mu_{\varphi\vee\psi}(z)$.

Nevertheless the most important goal we obtained is that this proposal makes possible, without introducing new concepts, but only using probabilistic tools, to solve problems containing randomness and fuzziness [61–64] such as the following (proposed to me by Zadeh) referable to a Bayesian updating of an initial probability [65]:

- *$A$ and $B$ are boxes, each containing 20 balls of various sizes. Most of the balls in $A$ are large, a few are medium and a few are small. Most of the balls in $B$ are small, a few are medium and a few are large. The balls in $A$ and $B$ are put into a box, $C$. What is the probability of drawing a ball from $C$ which is neither large nor small*?
- *Usually it takes Robert about an hour to get home from work. Usually Robert leaves office at about 5pm. What is the probability that Robert is home at 6:15pm?*

Our research in this ambit is not concluded: recently we proposed the same scheme, by using different measures of uncertainty, precisely coherent conditional plausibility [66] and coherent conditional possibility [67]. The interest for these studies comes from the necessity to manage real situations, where the information about the uncertainty of the elements $z \in C_Z$ is expressed by a non additive uncertainty measure. Nevertheless the results we obtained are not so close to the classical theory of fuzzy sets, such as those obtained by coherent conditional probability.

## 6 Mario Fedrizzi: Consensus modeling in group decision making

Most of the work carried on by the research team of the University of Trento leaded by Mario Fedrizzi (Silvia Bortot, Matteo Brunelli, Michele Fedrizzi, Ricardo Alberto Marques Pereira) was focused on the modeling of consensual processes in group decision making. It's well known that the notion of consensus plays a key role in modeling group decisions, and for a long time it was meant as a strict and unanimous agreement, however, since various decision makers have different more or less conflicting opinions the traditional strict meaning of consensus is unrealistic. The human perception of consensus is much "softer", and people are willing to accept that a consensus has been reached when most or the more predominant actors agree on the preferences associated to the most relevant alternatives. The "soft" meaning of consensus, advocated as realistic and humanly consistent, can lead to solve in a more constructive way group decision making situations by using modeling tools based on fuzzy logic. Accordingly,the problem of consensus reaching modeling in a fuzzy environment was addressed for first in [68,69], as based on individual reciprocal fuzzy preferences, [70–72]. The basic framework within which these consensus models have been developed can be depicted in the following way: There is a set of decision makers or experts who present their opinions concerning a set of alternatives and these alternatives may initially differ to a large extent. If the individuals are rationally committed to consensus, via some exchange of information, bargaining, etc. the individuals' opinions can be modified and the group may get closer to consensus. Here consensus is meant as the degree to which most of the individuals agree as

to their preferences concerning almost all of the relevant opinions. This degree of consensus takes on it values in the unit interval, and it's more realistic and human consistent than conventional degrees, mostly developed in the probabilistic framework. This approach was then extended in a dynamical setting in [73–75]. In this new model, the consensus reaching dynamics is generated starting from a combination of a soft measure of collective disagreement with an inertial mechanism of opinion changing aversion. It acts on the network of single preference structures by a combination of a collective process of (nonlinear) diffusion and an individual mechanism of (nonlinear) inertia. The overall effect of the dynamics is to outline and enhance the natural segmentation of the decision makers group into homogeneous preference subgroups. Each decision maker is represented by a pair of connected nodes, a primary node (dynamic) and a secondary node (static). The primary nodes form a fully connected sub network and each of them encodes the individual opinion of a single decision maker. The secondary nodes, on the other hand, encode the individual opinions originally declared by the decision makers and each of them is connected only with the associated primary node. The iterative process of opinion transformation corresponds to the gradient dynamics of a cost function depending on both the present and the original network configurations. The value of the cost function combines a measure of the overall disagreement in the present network configuration and a measure of the overall change from the original network configuration. The various interactions involving each node are mediated by interaction coefficients whose role is to quantify the strength of the interaction. The diffusive interaction between any pair of primary nodes is mediated by the interaction coefficient, whereas the inertial interaction between any primary node and the associated secondary node is mediated by the corresponding interaction coefficient. It turns out that the values of these interaction coefficients are given by the derivative of a scaling function. The diffusive component of the network dynamics results from the consensual interaction between each node and the remaining nodes in the network. The aggregated effect of these interactions can be represented as a single consensual interaction between the given node and a virtual node containing a particular weighted average of the remaining opinion values. The computer simulations of the model, particularly those in the fuzzy preference framework illustrated in [76], provided clear evidence that the fuzzy soft consensus model exhibits interesting non standard opinion changing behaviour in relation to the original crisp version of the model. In [77], in the framework of social networks analysis, we addressed the problem of consensus evaluation by endogenously computing the importance of the decision makers in terms of their influence strength in the network. To this aim, we start from centrality measure and combine it with the fuzzy m-ary adjacency relation approach. In this way, we introduce a flexible consensus measure that takes into account the influence strength of the decision makers according to their

eigenvector centrality. Moreover, we propose an optimization problem which determines the maximum number of the most important decision makers that share a fixed desirable consensus level. In [78] we investigated the relationship between the soft measure of collective dissensus and the multidistance approach to consensus evaluation. The novelty of the contribution consists in the introduction of a particular type of sum-based multidistance used as a measure of dissensus. This multidistance is characterized by the application of a subadditive filtering function whose effect is that of emphasizing small distances and attenuating large ones. An illustrative example is then developed in order to compare the new dissensus measure with the OWA-based multidistance obtained assuming that the weights are linearly decreasing with respect to increasing distance values.

## 7 Gabriella Pasi: my path in Fuzzy Logic

I read the first papers on Fuzzy Logic in 1990, when I was a researcher at the Italian National Council of Research (CNR). I came across the task of Information Retrieval well before Web search engines' birth, because I joined a research team that was working at defining and implementing an Information Retrieval System (Search Engine) for managing the scientific articles produced by CNR's researchers. In the phase of analysing the task of IR as well as the central role of the concept of relevance of documents to users' information needs, I had the opportunity of reading some papers written by Tadeusz Radecki and by Donald Kraft. In his seminal paper of 1979, titled "Fuzzy set theoretical approach to document retrieval", Radecki proposed a first generalization of the Boolean retrieval model [79]; this proposal was followed later by other authors. The main issue underlying this generalization was that Information Retrieval is a decision making activity which relies on subjective and often vague needs that an automatic system may only partially capture, and not exactly as in the Boolean model. The application of Fuzzy Set Theory (FST) and Fuzzy Logic to IR was initially proposed by several researchers such as Donald H. Kraft, Abraham Bookstein, Ducan Buell, Rita de Caluwe, Etienne Kerre, as an adequate formal framework for modeling the topical relevance of a document to a query as a gradual property, by thus allowing to shape IR systems that can rank documents based on their estimated relevance to user queries [80–82]. Based on this background in 1991 I co-authored (with Gloria Bordogna and Paola Carrara) a paper titled "Query term weights as constraints in fuzzy information retrieval", published in 1991 on the Information Processing and Management Journal [83]. A bit late, in 1992, at the First International Conference on Fuzzy Systems (Fuzz-IEEE) in San Diego we presented a paper titled "Extending Boolean Information Retrieval: a Fuzzy Model based on Linguistic Variables". This event was fundamental in determining a big part of the research activities I did undertake in subsequent years. At that conference I

met people with whom I started beautiful personal and professional relationships. I met Donald H. Kraft with whom Gloria and myself had an interesting working lunch in which we talked about the content of our proposals as well as the links to his works. I met Ronald Yager with whom I had the chance to discuss; as a consequence of the discussions with him, with Gloria we envisioned the usefulness of OWA operators for defining flexible query languages for IR systems, that we first proposed in 1995 in our paper titled "Linguistic Aggregation Operators of selection Criteria in fuzzy information retrieval" [84]. In San Diego I also met Patrick Bosc, Didier Dubois and Henry Prade, Jim Bezdek, Piero Bonissone and several other people who we would have seen and met in numerous subsequent events. Last but not least, I had the chance to meet Lotfi Zadeh, who became an important reference for the evolution of my research experience. What especially surprised me was his kindness, availability and curiosity in talking to me, in giving me advices. Overall I found a greater openness and enthusiasm of this scientific community with respect to other scientific communities. Several years have passed from that conference, and with the impressive development of the World Wide Web the era of big data has emerged, with the related need of defining systems able to provide users with an easy access to information relevant to their specific needs; this "old" research issue has become still more crucial. The uncertain and vague nature of the IR task has been deeply analysed from 1990 to now, and several approaches based on probability theory, machine learning techniques, and natural language analysis have been proposed. In this context, the application of Fuzzy Set Theory to Information Retrieval has been mainly aimed to the definition of retrieval techniques capable of modelling, at least to some extent, the subjectivity, vagueness and imprecision that is intrinsic to the process of locating information relevant to users' needs. Fuzzy Set Theory has proven to be a valuable means to define a variety of approaches, ranging from the definition of flexible query languages, fuzzy associative mechanisms, adaptive document's representations, and personalized systems [85–88]. In particular, Fuzzy Set Theory has been applied in the context of IR to the following main purposes:

- to deal with the imprecision and subjectivity that characterize the document indexing process, i.e., the process by means of which the content of a text is automatically synthesized and represented on the basis of keywords extracted from (or associated with) it;
- to capture and manage vagueness in query formulation: user requests are usually expressed by a few keywords, whose intended semantics can be disambiguated depending on both the user and query context, and whose importance in characterizing the search topics varies depending on several factor among which their specificity, their position within the query string, the collection searched, the user context, and other factors;

- to define "soft" associative mechanisms, such as ontologies, thesauri and algorithms for documents' clustering, which are often employed to extend the functionalities of an IRS.
- to define flexible decision strategies in meta-search engines (for fusing the ranked lists of documents retrieved by several search engines), or in distributed IR, for selecting the most adequate sources of information to a query based on their contents, reputation, user needs;
- to represent and query semi-structured information (XML document repositories) by allowing users to specify degrees of preference on the documents subparts they consider as the most significant to their needs, as well as to specify flexible constraints on both document structure and content.

To summarize, Fuzzy Set Theory has provided early ideas for modelling important features of IR systems, and it can still be promising for shaping search engines of the future. In particular, the notion of aggregation plays an important role in IR; the approaches that make use of the notion of aggregation rely on the interpretation of Information Retrieval (and of the related tasks)as a Multi-Criteria Decision Making (MCDM) problem, from various perspectives. The first and more straightforward perspective is to interpret the overall IR process as a MCDM process aimed at selecting the best alternatives (documents) based on the assessment of the performance of multiple criteria (the keywords specified in a user's query). Another important perspective is related to the assessment of the relevance of a document to a query, which is generally decomposed into the evaluation of the performance of several relevance dimensions (the considered criteria, e.g., topicality, novelty, popularity, recency, etc.). Another process that may require the application of appropriate aggregation operators is the indexing process, when applied to structured documents. Meta-search is another interesting task that can be seen as an instance of a Multi-Expert Decision Making problem, and that may rely on the appropriate choice of an aggregation operator. A quite interesting aspect implied by the previous interpretations of various phases of the IR process is that the choice of different aggregation operators can produce different results. Despite the potential impact of aggregation on the whole IR process, this aspect has not received the proper attention in the literature. Only recently some approaches have appeared demonstrating the importance of this issue, and its potential impact on the search process. Another interesting research issue where Fuzzy Set Theory could play a role is related to memes' identification and tracking. The word "meme" has been coined in 1976 by Richard Dawkins to indicate "the basic unit of cultural transmission or imitation": as well outlined by Susan Blackmore [89]: "Memes, like genes, are replicators, that is, they are information that is copied with variation and selection. Be-

cause only some of the variants survive, memes (and hence human cultures) evolve". As previously outlined, the Web constitutes a huge information repository, where several services and applications represent a rich soil for memes' growth and evolution. Let us think about blogs, e-mails, news resources and social networking services (like Twitter and Facebook). In the context of the Web, the concept of Internet Meme (e-meme) has been introduced as a unit of information (idea) that is replicated and propagated by people using one or more of the previously mentioned applications.

## 8 Aldo G. S. Ventre: A contribution to Fuzziness in Italy

I became assistant professor at the Institute of Mathematics of the Faculty of Architecture of the University of Napoli (now, University "Federico II") in 1969. In this Institute professors Giulio Andreoli and Angelo Fadini were studying logic of indeterminateness and they called "nebulosi", let say foggy, the induced sets. These researches developed further inspired by the paper "Les ensembles flous en linguistique" by Y. Gentilhomme [90]. The climate was favourable for a development of studies in fuzzy sets. Starting 1975 a strong curiosity led me to exchange ideas with Fadini [91,92] and Aldo De Luca on the topic of fuzzy sets. I was interested in the measure of fuzziness and defuzzification processes; I am grateful to them and Constantin V. Negoita, at that time professor in the University of Bucharest, for suggestions and advices. Antonio Di Nola, from the same Institute, and myself worked in the field of fuzzy entropy and induced structures [93–100] and other colleagues were interested in fuzzy sets: among them let me mention Rosanna Ambrosio, Virginia Cavaccini, Carmela Cella, Anna M. D'Aristotile, Ada Lettieri, Fortuna Liguori, Giulia B. Martini, M. Cristina Miglionico, Rossana Sarno, Salvatore Sessa. Starting the mid-eighties two branches were focused by Massimo Squillante and myself: decomposable measures w.r. to archimedean t-conorms [101], with extensions to classical topics of real analysis, e. g. the range of decomposable measures [100, 102–107] and the consistency for uncertainty measures in order to embed the various aspects of uncertainty in a unique context [108] with applications to financial laws [109] and economics (A.Maturo, Tofan, Ventre). This was a long term project in which also several young people joined. The interaction between the groups of Napoli and Trento, that stems in 1986 and continues until today, is fruitful, especially in the field of synthesis of judgements and multicriteria multiagent decision making, enhancing consensus in committees [110–112]. Further researches on the topic are carried out by Luciano Basile, Bice Cavallo, Livia D'Apuzzo, jointly with, Mario Fedrizzi, Michele Fedrizzi, Ricardo Pereira from Trento. Antonio Maturo, Barbara Ferri, Serena Sanseviero, Fabrizio Maturo, Simonetta Forcini from the University of Chieti-Pescara participate in the activities with Napoli and Trento developing also applications in social sciences and urban planning [113–117]. Relations with

statistics are studied in [118–120] We have fond memories of Pina Varone and Ada Lettieri died prematurely. During the years, relations and cooperation have been established with European and American Universities. Let us just mention the most rooted and lasting joint researches and initiatives with Peter Klement (Linz), Siegfried Weber (Mainz), Patrik Eklund (Åbo Akademi and Umeå) in different theoretical and applicative fields of mathematics and computer science.

## 9 Settimo Termini: How I began to do (and went on doing) a few fuzzy things

When I was asked to write something about the early years of fuzziness in Italy I thought that this could be an easy enterprise: just try to collect some early memories and put them in the right order, filling up, in some way, when and if necessary, some memory faults, something very probable since I am referring to the end of the Sixties of last Century, when I moved from the University to a CNR Research Laboratory outside University (a novelty for Italy, then) and begin to convert myself from a theoretical physicist to "something else" (it was, then, not clear what was this something else, and still continues to be unclear now[3].

However along the writing I realized that, if taken seriously, the enterprise was not so easy to do. But let's begin. It is easy to say when I heard about fuzzy sets for the first time. It was on 1969. Eduardo Caianiello - back from one of his scientific trips around the world - told me to look for a new interesting idea he had heard from Lotfi Zadeh. He did not have a copy of the paper, had not read it although he told me many details. His knowledge was simply a byproduct of a conversation with Zadeh. Luckily enough he remembered that had appeared on Information and Control "a few year before", so I was able to find it. In that period I had began to discuss scientific questions with Aldo de Luca, another physicist converted to "something else" and so we begin to discuss also Zadeh's paper. We moved along three different paths. The first was the one of "measuring the degree of fuzziness" [25] (the way along which we arrived at this idea is told in [122]), the second path regarded the analysis of the algebraic structures underlying this new idea [2]. But while analyzing

[3]Let me specify that - joking about myself - I refer to a big scientific question. The scientific and epistemological "status" of the so-called "information sciences", of which cybernetics was a prototype. A field that is still evolving and that has produced *wonderful* things in a very *chaotic* way - together with not equally valuable things, as frequently happens amid the chaos. Fuzzy sets are completely immersed in this context inside which they constitute a very meaningful presence. For a few personal remarks see [121]. In my view one cannot put order in this chaos by means of a sort of a "*rappel a l'ordre"* as was done by the artistic milieu at the beginning of XX Century. Information sciences (and, among them, the presence of fuzziness) ask for a radical *Re-Vision* of our traditional "habits of mind".

these purely "technical" aspects we felt that, having encountered a "qualitatively new" notion, a few epistemological reflections were unavoidable [123].

The first two papers appeared, so to say, in the "most adequate" journals. For the third one, we declined Lotfi's invitation to publish it in "Information Sciences". We were convinced that the "right" journal, in this case, was the old Italian *Scientia* founded at the beginning of the XX Century with the specific mission of presenting analyses of new scientific concepts[4].

We were not sure that our paper would be accepted in this venerable journal, and so were very proud when it, finally, appeared. Subsequently, however, we asked ourselves whether, by accepting Lotfi's invitation, the paper could have been read by a wider audience.

Subsequently, the initial ideas - especially those presented in [25] - were technically further extended and developed as, for instance, in [126–128] while the epistemological considerations of [123] were developed in two directions. The first (at the beginning, in a dialogue with Guglielmo Tamburrini) looked for possible paradigmatic connections of the background of the basic ideas of fuzziness with similar methodological approaches present in the development of cybernetics [129] (see also [130] for focusing the context and [121] for subsequent reflections on the context), Something that would be later masterly developed by Rudi Seising [131] (see also [132, 133]). The second tried to face the novelties involved in Lotfi's idea by analyzing them in a comparison with specific different approaches (like, for instance, Alternative Set Theory) [134–136]. Let me add that I looked also for connections with distinct (and distant) notions like "verisimilitude" by discussing with David Miller (these last attempts, I confess, did not produce meaningful results). Along these years, I had contacts and exchanges with a few colleagues and friends (e. g., Gianpiero Cattaneo, Tonino Di Nola, Aldo Ventre) and I also remember a very interesting (and *unusual*) discussion with Marisa Dalla Chiara at a Varenna school on the Foundations of Physics, along which she began to speak with me of the measures of fuzziness - a topic - she told me - which had heard by talking with Aldo de Luca (the paper had not yet been printed). "Unusual" since Marisa, at the beginning of our conversation, had not realized that I was involved in the topic.

After moving to University from the National Research Council, I remember with a great pleasure - during Perugia years (1987-90) - many discussions and scientific exchanges with Silvano Rivoira, Carlo Bernasconi, Giuliana Regoli and, especially, Giulianella Coletti and Romano Scozzafava (see [137,138]), just when the latter two began to pursue the brilliant path which finally produced such important and deep connections between De Finetti approach to probability and Zadeh's proposal (see [55]). Subsequently, from 1990, in Palermo

[4]A journal in which also Einstein and Russell (among many) had written exactly with this aim. (see, http://amshistorica.unibo.it/scientia). Unfortunately, Scientia ceased its publications in 1988 (see [124] and [125]).

when I moved here from Perugia, with the late Vito Di Gesù, an old date friend of mine, who involved me also in one of his periodic meetings [139].

No mention has been done of organizational activities; the contacts and exchanges mentioned above were, in fact, purely personal exchanges. If I can speak of an organizational activity (also this very limited indeed) this was oriented towards foreign contacts especially with Enric Trillas and all the things he begin to organize. Also in this case, I must confess, the driving role was his, not mine. Among these, I want to spend a few words on a Las Palmas Meeting (see [140]). This was, perhaps, the first (and unique) time in which I *did* try to promote a "public" confrontation on the problem of foundations and meaning of fuzzy sets. I thought it was very important to try establishing a link with other attempts at formalizing Vagueness. I worked hardly, from an organizational point of view, for having in Las Palmas the presence of two Czech logician (Jiri Becvar and Antonin Sochor) and asked to Siegfried Gottwald to tell the story of Klaua group. These were difficult things to obtain, before 1989. Unfortunately, however, it was not possible to have, after all these efforts, a true confrontation. The book cited above, published thanks to the efforts of Heinz Skala, fuzzily witnesses this effort.

Let me now explicitly refer to the difficulty I met in writing these pages. The request done to myself and others to present the "entrance" of fuzziness in Italy was not, I thought, concerned only to research and related results "in themselves" but presenting them - also - from an organizational point of view, describing how these new concepts begin to colonize the scientific environment of our Country. The fact that - although for purely causal reasons - Aldo and myself had been the forerunners in this new topic in Italy, would have implied - I thought when I begin writing these *souvenirs* - the duty of promoting and contributing to construct, here, some stable connecting network or national association (as happened elsewere). And *I didn't*. All the contacts listed above - and others, like the one with Silvano Di Zenzo, working at IBM Rome Scientific Center (a good occasion for a confrontation with industrial research, picking up similarities and differences) were not used by myself also in the direction of creating a research network. Was that a lost occasion? And, more importantly, does it point to some not irrelevant questions worth of some deepening? I - now - think that, following Ockham, also Associations "non sunt multiplicanda praeter necessitatem". In fact, the necessity for research is to present (and critically discuss) ideas, *new* ideas, their *theoretical* developments and their *applications* (two things really *strictly* connected, differently from what one usually thinks). And Italian scholars have provided ideas to the new field. The fact that this review is polyphonic, then, not only correctly describes the real situation but reflects what can be considered a right approach. However, let me add a few (personal) comments. One can work for organizational aspects only when it is clear *what* one aims to organize. In those remote years, I was trying to focus the innovative elements of this new field. The fail-

ure of Las Palmas attempt, together with the extreme difficulty of organizing it, is what definitely convinced me that the path towards a clarification could (or, perhaps, should) be pursued on at the level of purely personal contacts. This clarification would have been obtained by me 20 years later [141, 142].

Nothing has been, yet, said about the activities done after my return to Palermo University in 2009, after seven years spent in directing the CNR Institute of Cybernetics, now named to his founder, in which 40 years before I had heard by him the words *fuzzy set* for the first time[5]. Due, perhaps, to the long pause and thanks to the daily scientific interaction with Marco Elio Tabacchi, I was finally able to rethink at the questions posed by fuzziness in a fresh way focusing both on a critical analysis of the formal developments of the last decades and on a few questions posed by the use of this notion in soft sciences, the field that Zadeh had envisaged as the most fruitful for the application of his idea and which, in practice, had been very refractory [144, 145]. More details about these developments will be given by Marco with a fresh perspective.

## 10 Marco Elio Tabacchi: The transdisciplinary travel of Fuzziness

The first time I came across Fuzzy Sets I was a young and inexperienced student of Computer Science. As an aside of an image processing curriculum my professor spent a couple of hours introducing us to the mysteries of the idea of degree and Fuzzy arithmetic. The whole thing had a sort of secret sect bent, something like the tools that the powers to be don't want you to know and use – a probably unwitting attitude I have seen many times in the community and that has harmed more than helped. On an inexperienced and young student this approach undeniably had a major fascination, further built up by the reading of Kosko's Fuzzy Thinking [146] (of which I still use two chapters as student's material for my AI course, skimmed from the occasional psychobabble). I was fascinated by the elements of oriental philosophy behind the idea, but Vito Di Gesù, the person mentioned above, pointed me on the fact that fuzziness was perfect for describing phenomena that were very complicated to describe by means of traditional approaches and for operating on them without resorting to the intricacies of calculus. He showed me his ideas for a picture search engine based on fuzzy distances – years before google image. This idea resonated with my passion for images, as in a previous professional life I was a fine artist, and from this meeting started my first collaboration to a published scientific paper. The following years of research were dedicated to the appli-

[5] A period in which - due to the duty of following a chain of irrational "reforms" of CNR promoted by the *pro tempore* Government - I had hardly any possibility to do research work in whatever sense one choose to interpret the word. However, this hot climate contributed to force me to reflect on the conditions allowing to do research and guiding the way in which it can be directed and can usefully interact with the economic development of a Country [143].

cation of fuzzy methodologies to classification problems in arts and cognitive science. In the meantime I become the Scientific Director at *Istituto Nazionale di Ricerche Demopolis*, where staples of fuzzy logic such as CWW were applied to the analysis of natural language answers to questionnaires.

The untimely demise of Vito left me without a reference in the fuzzy world, but proving that sometimes lightnings can really strike twice, Settimo Termini of Fuzzy Entropy fame [25] was back in Palermo from his Naples' years (see above) and took me as a collaborator. Settimo's familiarity with the "two worlds" of human knowledge and his sensibility toward the foundational problems of uncertainty have added a new dimension to my research in Fuzzy Sets. We have discussed applications of Fuzzy Set Theory more as a step toward a more coherent theory of uncertainty, which considers not only degrees of ownership and composition thereof, but the inherent imprecision of (any description of) things. And extolled the pervasiveness of such concept not only in the more traditional human sciences, where this is somehow taken for granted, but also in hard science such as engineering and computing. These years spent in Fuzziness research have surely widened my views on uncertainty, and changed my expectations about what precise measurement can really yield – and why is a blessing. Not everything smells of roses, though. The extreme sectorialisation of Fuzziness into a discipline of systems has probably dissuaded practitioners of human sciences, which have often snubbed it, from taking an interest despite there being a willingness for a joint effort. The possible role of Fuzziness as a methodological bridge between the hard sciences and humanities was as such neglected. We have worked long and hard to try and reaffirm this significative aspect of fuzziness [141, 145, 147, 148] still without a lot of luck. Nonetheless, the deep relationship between Fuzziness and the cognitive aspects of human nature survived and resurfaced in a number of methodologies from the more general context of Computational Intelligence (CI). I have married my interests in fuzziness and cognitive science [132, 133], by working on how human performances in a different set of tasks can be reproduced and helped by Soft Computing and Computational Intelligence methodologies. Some traces of this research path can be found in [149–152]. A recent development of this research have discussed the role of transdisciplinarity and the relationship between cognitive science and cybernetics at a foundational level [121, 153], something which directly evokes the role of fuzziness in modern science. This has more recently lead to a discussion on how the same techniques can be fruitfully employed in the ongoing transformation of the idea of smart city (sensors everywhere) in cognitive city: colloquium everywhere, between humans and machines, machines and machines, in the spirit of Wiener [154, 155].

From my point of view (both physical and intellectual), the outlook for fuzziness in Italy is good. The variety of approaches that this contribution is testimony of has gifted Italy with a varied perspective, where fuzziness, more

that in other realities, has interacted and integrated with different disciplines in a coherent transdisciplinary fashion. During Gabriella Pasi's presidency of EuSFLaT, the Italian community of the association has grown both in numbers and participation. This has sublimated the lack of a strong national association (despite IEEE Fuzzy), and has improved the exchange between Italian scientists, as anyone who recently attended one of EuSFLaT conferences can testimony. I personally hope for the next fifty years for a complete dissolution of fuzziness, cognitive science, and all the other interested disciplines into a unique transdisciplinary science, beyond academical squabbles and disciplinary boundaries.

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# The first steps in fuzzy set theory in France forty years ago (and before)

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**Abstract** At the occasion of the fiftieth anniversary of the founding article "Fuzzy sets" by L. A. Zadeh, we briefly outline the beginnings of fuzzy set research in France, taking place some ten years later, pointing out the pioneering role of Arnold Kaufmann and few others in this emergence. Moreover, we also point out that the French counterpart of the name "fuzzy set" had appeared some 15 years before Zadeh's paper, in a paper written in French by the very person who also invented triangular norms in the 1940's.

*This paper is a translated and expanded version of a conference paper [31].*

## 1 Before the beginning

Strangely enough, the phrase "ensembles flous" (the French translation of "fuzzy sets") first appeared in a paper published in French in the Comptes Rendus of the French Academy of Sciences in 1951 [74] by Karl Menger (1902-1985). He was an Austrian mathematician [96] who emigrated to the USA before the second World War. He is the son of the well-known economist Carl Menger (1840-1921) himself one of the fathers of the theory of subjective utility value. Karl Menger was an active member of the Vienna Circle; later he was at the origin of triangular norms with the paper "Statistical metrics" [73], where triangular norms emerge in stochastic geometry from the generalization of the classical triangle inequality when distances between two elements of a metric space are represented by probability distributions rather than by numbers. His spectrum of interest, which was very wide [76], included logic. He especially proposed a "logic of the doubtful" where (italics are from Menger himself):

> we divide the propositions into three mutually exclusive classes of modality: $\mu_+$ consisting of the *asserted*, $\mu_0$ consisting of the *doubtful*, $\mu_-$ consisting of the *negated* propositions. $[\cdots]$ In contrast to the traditional 3-valued logic, *the modality of a compound is not determined by the modalities of the components*." [72]

He was thus making it clear very early that uncertainty is not compositional.

What is truly remarkable in Menger's 1951 paper is not only the use of the French counterpart to "fuzzy sets", but also its application to a notion closely related to Zadeh's idea: in fuzzy set terminology, the paper is about max-product transitive fuzzy relations! However, in Menger's paper, we can read (p. 2002):

> Nous appellerons cette fonction même un *ensemble flou* et nous interpréterons $\Pi_F(x)$ comme la probabilité que $x$ appartienne à cet ensemble. Si $\Pi_F$ ne prend que les valeurs 1 et 0, il s'agit essentiellement d'un sous-ensemble de $U$ au sens classique et nous parlerons d'ensemble *rigide*. [1]

Moreover, Menger was soon aware of the emergence of fuzzy sets since one year after the publication of Zadeh's seminal paper [104], he wrote [75]:

> In 1951, I suggested that, besides studying well-defined sets, it might be necessary to develop a theory in which the element-set-relation is replaced by *the probability an element belonging to a set*.

[1] In English: "We shall call such a function a *fuzzy set* and we shall interpret $\Pi_F(x)$ as the probability that $x$ belongs to this set. If $\Pi_F$ only takes the values 1 and 0, it is essentially a subset of $U$ in the classical sense and we shall speak of *crisp* set."

> In a Paris note [Ref] [2], I called such an object, in contrast to an ordinary or rigid set, *ensemble flou* (= hazy set)." In a slightly different terminology, this idea was recently expressed by Bellman, Kalaba and Zadeh [Ref][3] under the name of *fuzzy set*. (These authors speak of the degree rather than than the probability an element belonging to a set.)

Thus, the distinction between probability and degree of membership was very clear for Menger from the beginning; see [24, 99] for further discussions.

A worth noticing coincidence took place on May 28, 1951, the day when the French mathematician Arnaud Denjoy (1884-1974) transmitted Karl Menger's communication on "ensembles flous" to the French Academy of Sciences. Indeed, Arnaud Denjoy also transmitted on the same day a communication by Gustave Choquet [15] whose abstract was

> En vue d'une théorie des fonctions non additives d'ensembles, on définit et l'on étudie la classe des sous-ensembles des espaces séparés engendrée à partir des compacts par réunions ou intersections dénombrables et par applications continues"[4]

Thus on the same day where the phrase "ensemble flou" appeared, elements towards the theory of capacities (i.e., "fuzzy measures" in Sugeno's terminology [100]) and Choquet integrals [16] (which would appear later as the quantitative counterpart of Sugeno integrals) were also presented. The fuzzy future thus began on May 28, 1951, even if several decades and a significant research effort would still be necessary before the full landscape could be put together.

Besides, another piece of early work, apparently written independently from Zadeh's pioneering paper, is also worth mentioning. It is an article in French published in 1968 by a French linguist, Yves Gentilhomme (b. in 1920) in a Romanian journal [40]. In this paper, Gentilhomme calls "ensemble flou" a nested pair of subsets, one gathering what he regards as "the central elements", while the second larger subset also includes "peripheral elements". Gentilhomme motivates his proposal by an example of "hypergrammaticality" in texts (illustrated by a poem by Alphonse Allais where the author intentionally makes an abusive use of the imperfect tense of the subjunctive mood in order to produce a comical effect) and by an example of more or less credible words in French built from the same root. Then Gentilhomme provides a formal set-theoretic apparatus for combining his "ensembles flous", and he proposes to assign a degree of membership equal to 1/2 to peripheral elements

[2] Reference [74].

[3] R. Bellman, R. Kalaba, L. Zadeh. Abstraction and pattern classification. J. of Mathematical Analysis and Applications, 13 (1), 1-7, January 1966.

[4] In English: "In view of a theory of non additive set functions, one defines the class of the subsets of separated spaces, generated from compacts by denumerable unions or intersections and by continuous mappings."

(those that are not central). It is in the 1974 pioneering research monograph by Negoita and Ralescu [81] (who were working in Romania at that time) that Gentilhomme's paper is first reported and put in relation with Zadeh's work; "ensembles flous" are translated by "flou sets" in the English version of the book the year after [82].

## 2 Arnold Kaufmann

Edwin Diday (b. 1940) seems to have published the first journal paper in France influenced by the fuzzy set idea [22] in 1972. The paper presents a new approach to (fuzzy) clustering, called the dynamical cloud method (in French, "méthode des nuées[5] dynamiques"), and cites Zadeh [104] and Ruspini [92].

However, it is Arnold Kaufmann (1911-1994) [28, 29] who unquestionably introduced fuzzy set theory in France. He was an applied mathematician, author, or co-author of a long series of books covering many areas in engineering mathematics, including automatic control and operations research. His books, many of which were translated into English, were not only covering standard applied mathematics, but also many advanced topics in relation with current research at that time [19–21, 41–46, 48–50, 59–63][6]. As he was in contact with Lotfi Zadeh, he heard about fuzzy sets very early, and was quickly enthusiastic about this challenging way of thinking. He was the first in the world to publish a monograph on the theory of fuzzy (sub)sets in 1973 [51]. It was translated into English two years later [54][7]. The first 1973 volume was soon followed by three other ones [52, 53, 55], and by a book of exercises [58] (with Michel Cools and Thierry Dubois). Altogether, this series of five volumes mainly cover fuzzy set theoretic operations, fuzzy relations, and their applications to many fields: classification and pattern recognition, automata and systems, multicriteria decision, as well as linguistics, logic, topology, matroïds, etc. These books, and in particular the first one, had a great impact on the dissemination of fuzzy set theory in France. Continuing to write books on fuzzy logic-related topics, his strong interest for the topic never waned until the end of his life.

Let us quote the last paragraph of the conclusion of his first fuzzy set book [51]:

> Je voudrais exprimer un souhait très sincère. Je voudrais que mes lecteurs, initiés et intéressés par mon modeste travail, puissent aller plus loin, beaucoup plus loin, encore plus loin. Les sciences

[5] The use of this word , which means "clouds" reminds us that at the beginning Zadeh was hesitating between the words "fuzzy" and "cloudy": Indeed, he wrote in 1962: "we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions." in [103]

[6] This list by no means claims to exhaustiveness!

[7] This more theoretical book, first written in Romanian in 1974 [81] would appear in English also in 1975 [82].

> humaines ont besoin d'une mathématique appropriée à notre nature, à nos attitudes floues, à notre comportement nuancé, à nos dosages, à nos critères multiples. Si ce premier livre est suffisamment stimulant, de nombreux articles, concernant les aspects théoriques ou les applications, seront publiés par des lecteurs; des livres concurrents verront le jour. Tout ceci pour l'amélioration rapide de nos méthodes en vue d'aborder les sciences humaines."[8]

Beyond the fact that enthusiasm and generosity permeate this text, and the correctness of this prediction, it is also worth noticing that Kaufmann was considering that the applications of fuzzy set theory would be human-oriented sciences, while this is not so clear as of to-day. We have to remember that Kaufmann was writing this text at a time where information processing and artificial intelligence were still in infancy.

At the time when he got acquainted with fuzzy sets, Kaufmann was deeply interested in methods for helping creativeness [47], a topic on which he published a book later in 1979 [56]. So, it was one of the first areas where he considered applying fuzzy sets [1, 55, 57], for which he proposed a lattice and fuzzy relation-based approach, in the spirit of ideas and methods preiously advocated by Abraham Moles (1920-1992), an engineer by training, then a philosopher, working on the sociology and psychology of information and communication sciences [77–79] [80]. Kaufmann's approach to creativeness was also developed by his co-authors Michel Cools and Monique Peteau [17].

## 3 Elie Sanchez - Claude Ponsard - Robert Féron

The first three main followers of Arnold Kaufmann in France in the mid-1970's are Elie Sanchez, Claude Ponsard, and Robert Féron.

Elie Sanchez (1944-2014) [12, 98, 102] was the first in France, in 1974 in Marseilles, to defend a thesis on fuzzy set methods. His thesis is a landmark piece of work on fuzzy relation equations, which contains important results on the solving of these equations [94]. This research was motivated by an attempt at a mathematical formalization of medical diagnosis. It had a very strong influence on the development of fuzzy set methods worldwide.

Claude Ponsard (1927-1990) [9, 27, 39], working in Dijon, started to propose in 1975 to apply fuzzy sets to various problems in economics [87, 88]. He

[8] "I would like to express a very sincere wish. I would like that my readers, taught and interested by my modest work, go farther, much farther and farther. Human-oriented sciences need a kind of mathematics that fits our nature, our fuzzy attitudes, our nuanced behavior, our balanced judgments, our multiple criteria. If this first book is sufficiently stimulating, many articles, dealing with theoretical or practical aspects, will be published by my readers; concurrent monographs will appear. All this, for the fast improvement of our methods for coping with human-oriented sciences."

then soon after led a small group of researchers on these questions, including Bernard Fustier [38] and Régis Deloche [18].

Robert Féron (b. 1921) [5] is a statistician working in econometrics in Lyon. He was the first to properly provide a theoretical basis for the study of fuzzy random sets and to advocate their interest [33–36]. His writings, mostly in French, and mostly published in a journal having a limited circulation, would remain unfortunately largely ignored in the English American world. Let us also point out, on the same kind of topic, and published in the same journal, a paper by Robert Fortet and Mehri Kambouzia [37]. Also related to this trend of research are works by Israël-César Lerman (b. 1940) on fuzzy classification [65] and by Jean-François Martin on fuzzy histograms [67,68].

We should also mention the two pioneering papers by Jean-Pierre Aubin (b. 1939) on game theory with fuzzy cores (the set of multistrategies that are not rejected by any coalition) published at that time [3,4].

## 4 1974-1976: The pivotal years

From 1974-1976 on, the number of young French researchers interested in fuzzy sets started to increase (even if the topic was not very popular and remained highly controversial in academia especially in France (many scholars were considering that it was not a "serious" topic to work on - partly because of the name -, and whose connection / difference with probability was unclear). Let us provide a list of persons in France who began to use fuzzy sets in their works at that time:

- Bernadette Bouchon, a member of the team headed by Claude-François Picard (1926-1979) [85] started working on fuzzy questionaires [10],

- Jacques Brémont (1938-2014) defended his thesis on the use of fuzzy sets on speech recognition in 1975 [13] under the supervision of Michel Lamotte [14, 70]. Altogether with Gérard Hirsch (b. 1938) [7] they later on formed an active research group on fuzzy set methods in Nancy for about two decades.

- The first French works on fuzzy systems were initiated by Pierre Vidal [69] in Lille in 1974, together with Noël Malvache (1943-2007) who then continued to work on fuzzy rule-based controllers with Didier Willaeys in Valenciennes [101].

- the year 1975 saw the publication of the first fuzzy set papers by two French pure mathematicians, Daniel Ponasse [86] and S. Ribeyre [91]. The former, once back in France, launched a seminar on "Fuzzy Mathematics" in Lyon, which would become very productive in the late 1970's and the 1980's. This group included Achille Achache (b. 1934), Nicole Blanchard, Odile Botta, Josette and Jean-Louis Coulon, Marianne Delorme, and Christiane Dujet. Let us also mention Michel Eytan [32] on this mathematical side.

Although the following people have been more briefly involved with fuzzy sets, one may still mention:

- in automation of production processes, the thesis of Moncef Ben Salem (1953-2015) [8] where a fuzzy multicriteria automatic decision-making procedure is proposed for determining the sequencing of operations accomplished by a machine tool. Later, Ben Salem became minister of Higher Education and Scientific Research (2011-2014), after the Tunisian revolution. Lucas Pun [90] was one of the very first in France to foresee the potential interest of fuzzy sets in the modeling of production processes.

- Jean-Marc Adamo (b. 1943) [2] started working in the second half of the 1970's for some years on dynamical systems and then on fuzzy programming languages.

- Jean-Philippe Massonie [71] in Besançon was the first in France to foresee the potential interest of fuzzy sets in geographical modeling, thus initiating a line of research that still exists in France.

What is more unexpected is that fuzzy sets were also a source of inspiration in the French avant-garde literature. The French novelist Claude Ollier [83], a writer close to the "Nouveau Roman" movement, seems to have met Arnold Kaufmann in September 1970, at a meeting about creativeness in art and science [1], where Kaufmann already spoke about fuzzy sets. In this French novel with an English title "Fuzzy sets", the author plays with the roles oDub78f the protagonists and the reader in the story, as well as with the display of the text on the pages. The novel was reprinted with a slightly less exotic page display two decades later [84]. Let us mention a more classical writer, Jacques Laurent, who published a novel also with the title "Les sous-ensembles flous" (this time in French) a bit later in 1981 [64]. In this book, the fuzzy set idea applies at several levels to the links between the characters and the forces that drive them. Let us also mention in computer art the research of Alex Makarovitsch on visual fuzziness [66].

Lastly, it is in 1976 that the authors of this note produced their first (handwritten!) research report of [25] (now indexed by Google Books). This was mainly a survey and a status report. Our first published contributions only appeared one year later [23, 26, 89].

## 5 Conclusion

This note is an attempt at offering a short overview of the first years of research in France regarding fuzzy sets and their applications. Only references from mid-seventies and before have been reported, without mentioning further developments of the works of the authors cited. In fact, some authors have encountered fuzzy sets very briefly in their research in this time period, while others have continued to contribute to fuzzy sets for several decades.

As shown by this brief overview, fuzzy set research in France (see [11, 30] for general overviews), starts in the years 1973-1976, and immediately deals with very different issues. They are led by researchers relatively isolated from

one another, who often face suspicion, negative critique, and sometimes disparagement and bashing from their academic colleagues. Nevertheless, those times were more open-minded than the present period that seems to be under the tyranny and normalization of citation rates and impact factors!

It is also worth noticing that the first works rely mainly on fuzzy set operations and on fuzzy relations, and that many important notions have no role in these works, even if they already exist as the extension principle [104], or the notions of fuzzy measures and fuzzy integrals in the sense of Sugeno [100][9]. It is a matter of facts that some crucial developments would only appear a bit later, such as possibility theory [105], or the linkage between fuzzy set connectives and triangular norms, originally introduced in the study of probabilistic metric spaces, whose father was precisely Karl Menger, the man who first used the phrase "ensemble flou"!

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# Fuzzy Sets and Fuzzy Logic in Austria

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**Abstract** We sketch the development of fuzzy sets and fuzzy logic in Austria during the last 50 years which started, after some early traces, in 1976 in Linz with my own work and in Vienna with Klaus-Peter Adlassnig's work. Therefore we first discuss the history of our research group at the Johannes Kepler University Linz and at the JKU-Softwarepark Hagenberg. Next we have a closer look at the developments at the Vienna University Medical School (group of Klaus-Peter Adlassnig), at the logic group at the Vienna University of Technology (in particular the contributions of Agata Ciabattoni), and at the work of Reinhard Viertl in statistics. Finally we present some achievements made at the University of Innsbruck (Michael Oberguggenberger).

## 1 Early Traces

Before publishing his first paper on fuzzy sets [1], Lotfi A. Zadeh had been an influential pioneer of system theory. His monograph on linear system theory [2] (jointly written with Charles A. Desoer) was a standard reference in this rather young field which had emerged, together with some other areas like cybernetics, from roots in electrical engineering, logic and mathematics, and which was rather interdisciplinary in nature.

It is therefore no big surprise that the first researchers in Austria who paid attention to fuzzy set theory came from system theory and cybernetics: Franz Pichler from the Johannes Kepler University Linz (JKU) who was the first Austrian to hold a professorship for system theory, and Robert Trappl, professor of cybernetics at the University of Vienna who organized a series of bi-annual conferences in Vienna, the *European Meeting on Cybernetics and Systems Research* (EMCSR) which always included sessions on fuzzy sets and systems chaired by Christer Carlsson (Åbo).

Both Franz Pichler and Robert Trappl were in close touch with many international researchers, including Lotfi A. Zadeh (Berkeley), George J. Klir (Binghampton, NY), Ronald R. Yager (New Rochelle, NY), and Heinz Schwärtzel (Munich), to name just a few. Several of these contacts have been made possible by the *International Institute for Applied Systems Analysis* (IIASA), located on the neutral ground of the Castle of Laxenburg near Vienna, which had been founded in 1972, i.e., during the Cold War, at The Royal Society in London as an interdisciplinary, non-governmental research institution by the United States, the Soviet Union and ten other countries from the Eastern and Western blocs.

## 2 Activities in Upper Austria

As already mentioned, at the Johannes Kepler University Franz Pichler took an early interest in fuzzy sets, discussing this topic in his lectures and seminars. In the fall of 1976, I was asked by my department boss, Peter Weiß, to have a look at a paper of Ulrich Höhle [3] dealing with measures on fuzzy sets, which just had appeared (in German) in the *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* (renamed in 1986 into *Probability Theory and Related Fields*).

I got interested in the idea of fuzzy sets, studied the existing literature and started to get in touch with some colleagues, in particular with Ulrich Höhle (Wuppertal) and Robert Lowen (Brussels). In 1978, at the occasion of the Fourth European Meeting on Cybernetics and Systems Research (EMCSR) organized by Robert Trappl and Franz Pichler at the JKU, I had a chance to meet for the first time L. A. Zadeh who gave the opening lecture of the conference. After that meeting he invited me to spend some time as a visiting scholar at the Computer Science Division, Department of Electrical Engineering and

Computer Science and at the Department of Mathematics of the University of California, Berkeley.

Supported by a Postdoctoral Research Exchange Grant of the Max Kade Foundation Inc., New York, and by a Fulbright travel grant, I visited Berkeley for six months (February till July) in 1979. During my whole visit, I was in close touch with L. A. Zadeh and his international PhD students at that time, Piero Bonissone (Mexico), Christian Freksa (Germany), Ramon Lopez de Mantaras (Spain), and Richard Tong (United Kingdom). I also met several researchers from the fuzzy community working in the United States or visiting Berkeley, including Masao Mukaidono (Kawasaki), Hung T. Nguyen (Amherst, MA), Henri Prade (Toulouse), Dan Ralescu (Bloomington, IN), Azriel Rosenfeld (College Park, MD), Enrique H. Ruspini (Menlo Park, CA), Patrick Suppes (Stanford), and Ronald R. Yager.

My research stay in Berkeley was quite an important contribution to the habilitation I passed in 1981 at the Johannes Kepler University, where I later was appointed Associate Professor (1993) and Professor (1998). Over the years I held visiting professorships at the Universität für Bildungswissenschaften Klagenfurt (1982), the University of Cincinnati (1983), and the Università degli Studi di Trento (1993). Other long-term stays as a visiting research fellow included the Université Aix-Marseille II (with Elie Sanchez, 1981 and 1988) and the Tokyo Institute of Technology (with Michio Sugeno, 1987), and there were a number of shorter research visits, mainly in Europe and Israel.

In 1990 I had the luck to become a partner in Franz Pichler's research project "CAST for complex systems" which was sponsored by Heinz Schwärtzel from the SIEMENS AG (Munich). This allowed me to found the *Fuzzy Logic Laboratorium Linz-Hagenberg* (FLLL) which started in October 1991 with an inaugural lecture by Takeshi Yamakawa (Iizuka).

Two years earlier, Bruno Buchberger from the Research Institute of Symbolic Computation (RISC) at the JKU had established the *JKU-Softwarepark Hagenberg*, in a small village with little more than 1000 inhabitants, less than 20 miles north of Linz. The FLLL moved as one of the first research institutions into this new technology park, which over the years turned out to become a major success story in mathematics and computer science, both from a theoretical and applied point of view [4]. One of the highlights in the first years was L. A. Zadeh's keynote speech "Fuzzy Logic—A Key Technology for the Future" in October 1992.

The FLLL started to grow and became well-known both for its theoretical research and its applications for the industry. We were and are involved in several projects at the European, national and regional level, including two competence centers within the COMET program of the Austrian government. Starting in 1990 we had a rather big number of visitors (mostly from former COMECON countries). These foreign students were supported by European programs and networks such as ERASMUS, COMETT, LEONARDO, COST and,

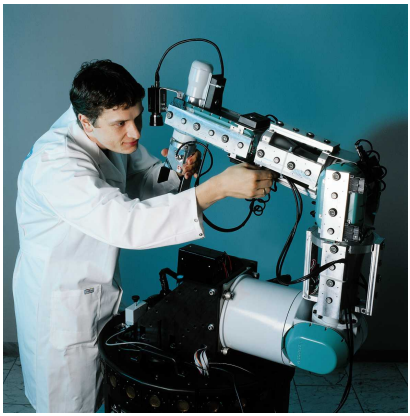

Figure 1: Kurt Häusler and his robot *Amadeus*: developed at Profactor in connection with his PhD thesis "Fuzzy motion planning for a human-arm-like redundant manipulator" (2002) © Profactor GmbH

most of all, CEEPUS. Stephen E. Rodabaugh (Youngstown, OH), Dan Butnariu (Haifa), László T. Kóczy (Budapest), Frank Klawonn (Braunschweig), Radko Mesiar (Bratislava), and Endre Pap (Novi Sad) served as visiting professors at JKU, providing new ideas and research results for our students. Over the years we supervised 30 master and 20 PhD theses (being involved in some 15 defenses of PhD theses at other universities).

The FLLL also was and is involved, in close cooperation with industrial partners, in applied projects in areas like image and signal processing, optical fault detection, prediction in and control of complex systems (including robotics), as well as data mining, machine learning, and knowledge-based modelling (see, e.g., [5]). Our list of industrial partners includes, among others, HILTI (Liechtenstein) and Heidenhain (Germany), as well as a number of Austrian companies like AVL List, ekey, KEBA, Profactor, SCA Laakirchen, Sony DADC, TRUMPF Maschinen, Uni Software Plus, and the Verbundkonzern.

A major contribution to the development of mathematical aspects of fuzzy logic was the organization of so far 36 *Linz Seminars on Fuzzy Set Theory* between 1979 and 2016, which emphasize the development of mathematical aspects of fuzzy sets by bringing together researchers in fuzzy sets (including young and promising colleagues) and established mathematicians whose work outside the fuzzy setting can provide direction for further research. The philosophy of the seminars has always been to keep them deliberately small and intimate so that informal critical discussions remain central. A broad international representation and a mix of pure and applied interests are other essential features. There are no parallel sessions, and there is ample time for

discussion of each presentation, with followup round tables for discussion of open problems and issues raised in the talks.



Figure 2: Participants at the $25^{\text{th}}$ Linz Seminar on Fuzzy Set Theory "Mathematics of Fuzzy Systems" (February 2004) © María Ángeles Gil

Based on an initiative of U. Höhle, R. Lowen and myself, in September 1979 the *International Seminar on Fuzzy Set Theory* with eight participants from France, Belgium, Germany, Austria and the United States took place for the first time in Linz. The seminars until 1987 were jointly organized by the three founders. Then a permanent Program Committee was established, and a specific topic was chosen for each seminar. The following colleagues served as (co-)chairpersons of this Program Committee: Ulrich Bodenhofer (2009), Dan Butnariu (Haifa; 1998), Petr Cintula (Praha; 2010), Bernard De Baets (Gent; 2009, 2016), Phil Diamond (Brisbane; 2002), Didier Dubois (Toulouse; 1996, 2007, 2011), Tommaso Flaminio (Varese; 2014), János Fodor (Budapest; 2006), Lluís Godo (Barcelona; 2014), Siegfried Gottwald (Leipzig; 1997, 2005, 2014), Michel Grabisch (Paris; 2011), Petr Hájek (Praha; 2005), Ulrich Höhle (Wuppertal; 1992, 2005, 2012), Erich Peter Klement (1993), Wesley Kotzé (Grahamstown; 1994), Radko Mesiar (Bratislava; 2003, 2007, 2011, 2013, 2016), Endre Pap (Novi Sad; 2004, 2013), Henri Prade (Toulouse; 1996), Stephen E. Rodabaugh (Youngstown, OH; 1989, 1999, 2008), Marc Roubens (Liège; 2001, 2006), Susanne Saminger-Platz (2009, 2016), Lawrence N. Stout (Bloomington, IL; 2000, 2008, 2010, 2012), Llorenç Valverde (Palma de Mallorca; 1990), and Siegfried Weber (Mainz; 1988, 1991).

Several edited volumes containing selected and expanded papers have been published after some of the seminars [6–10], as well as 11 special issues of *Fuzzy Sets and Systems* and one special issue of *Quaestiones Mathematicae*.



Figure 3: Presentation of the title *Doctor honoris causa* to Lotfi A. Zadeh at JKU (2003)

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In 1993, Wolfgang Slany (Technical University Vienna) and myself organized the *Eighth Austrian Artificial Intelligence Conference* (FLAI '93) devoted to "Fuzzy Logic in Artificial Intelligence". L. A. Zadeh gave the opening lecture on "The Role of Fuzzy Logic and Soft Computing in the Conception and Design of Intelligent Systems", and the proceedings were published by the Lecture Notes of Artificial Intelligence [11].

I was also involved in the organization of several other conferences, including the second and the sixth "IFSA World Congress" (Tokyo, 1987 and São Paulo, 1995, respectively), a series of workshops "Current Issues in Fuzzy Technologies" (CIFT) in Trento (co-chair with Mario Fedrizzi and others, 1991–1995), and the "International Conference on Fuzzy Set Theory and its Applications" (FSTA) in Liptovský Ján (Co-Chairman of the Program Committee with, among others, Radko Mesiar and Susanne Saminger-Platz, 1996–2014).

In 1997 I was in the first group of researchers who were named Fellow of the International Fuzzy Systems Association (IFSA). And after my retirement Susanne Saminger-Platz and Radko Mesiar dedicated to me the volume "On Logical, Algebraic, and Probabilistic Aspects of Fuzzy Set Theory" [12].

My main research interests can roughly be divided into two areas. The first one deals with monotone set functions (generalized measures) and integrals

based on them, including applications to probability theory, game theory and statistics. In this area, papers in international journals, edited volumes and conference proceedings were co-authored with (in chronological order) Robert Lowen, Madan L. Puri, Dan Ralescu, Ulrich Höhle, Siegfried Weber, Dan Butnariu, Radko Mesiar, Endre Pap, Anna Kolesárová, Andrea Stupňanová, and Jun Li. The monograph on triangular norm-based measures and games with fuzzy coalitions written together with D. Butnariu [13] deserves special mentioning.

The second area of interest deals with some of the logical and algebraic foundations of fuzzy set theory, with a special emphasis on distinguished aggregation functions such as triangular norms and copulas. I had very fruitful research cooperations with many colleagues from abroad, including Dan Butnariu, Radko Mesiar, Endre Pap, Bernhard Moser, Mirko Navara, László T. Kóczy, Anna Kolesárová, Petr Cintula, Fabrizio Durante, Jose Juan Quesada-Molina, Peter Sarkoci, Carlo Sempi, Susanne Saminger-Platz, Andrea Stupňanová, János Fodor, Maddalena Manzi, and Fabio Spizzichino. Our monograph on triangular norms, co-authored by R. Mesiar and E. Pap [14] has been especially well-cited.

On January 8, 2003, the Johannes Kepler University awarded the title *Doctor honoris causa* to Lotfi A. Zadeh, in recognition of his outstanding contributions to systems theory and fuzzy logic. As chairman of the academic senate of our university at that time, it was my privilege and honor to deliver the laudatio at this occasion and to hand over to him the corresponding certificate.

The following day we moved again to Hagenberg to celebrate ten years of FLLL in Hagenberg with a small symposium "Fuzzy Logic. Eine Technologie setzt sich durch!", and Lotfi A. Zadeh spoke about "The Role of Fuzzy Logic Today" (the other speakers were Klaus-Peter Adlassnig, Ulrich Bodenhofer, Kurt Häusler, László T. Kóczy, and myself).

Edwin Lughofer, a senior member of FLLL, specialized after his PhD in machine learning and evolving fuzzy systems [15–17] and served as the chairman of the "IEEE Evolving and Adaptive Intelligent Systems Conference 2014" in Linz.

Four (former) members of FLLL passed their habilitation so far:

Ulrich Bodenhofer who worked as Scientific director of the Software Competence Center Hagenberg (2002–2006) and served as president of the European Society for Fuzzy Logic and Technology (EUSFLAT, 2005–2009), now teaches at the Department of Bioinformatics of JKU.

Fabrizio Durante moved to the Faculty of Economics and Management Free University of Bozen-Bolzano.

Susanne Saminger-Platz who had passed her PhD with the highest (Austrian) distinction ("Promotio sub auspiciis praesidentis rei publicae") visited the the Università del Salento, Lecce (with Carlo Sempi, 2006–2007) and succeeded me in 2014 as head of the Department od Knowledge-Based Mathematical System at JKU, where also Thomas Vetterlein continues to work.



Figure 4: Participants at the Symposium "Fuzzy Logic and Related Aspects" at the occasion of my retirement (October 2014) © Robert Pollak

## 3 Activities in Vienna

In 1976, after moving to the Department of Medical Computer Sciences, University of Vienna Medical School, Klaus-Peter Adlassnig became interested in fuzzy sets which he used in his PhD thesis at the Vienna University of Technology "Ein computerunterstütztes medizinisches Diagnosesystem unter Verwendung von fuzzy Teilmengen" ("A computer-assisted medical diagnostic system using fuzzy subsets", completed 1983).

From 1984 until 1986 he worked with Lotfi A. Zadeh as a visiting research fellow at the Computer Science Division, Department of Electrical Engineering and Computer Science, University of California, Berkeley, supported by a fellowship of the Max Kade Foundation Inc., New York.

Later on, he was a visiting associate professor at the Department of Medicine, Section on Medical Informatics, Stanford University School of Medicine (1993, supported by a Fulbright Fellowship), again a visiting scholar at the Department of Electrical Engineering and Computer Sciences, Computer Science Division, Berkeley Initiative in Soft Computing (BISC), University of California, Berkeley (2000), a visiting research fellow at the Department of Computer Science, Meiji University, Kawasaki (2005), and a visiting scientist at the Clinical Decision Making Group, Computer Science and Artificial Intelligence Laboratory (CSAIL), Massachusetts Institute of Technology (MIT), Cambridge, MA (2008).

At the University of Vienna Medical School he passed his habilitation in



Figure 5: Klaus-Peter Adlassnig (left, at the HL7 Meeting (2012) in Vancouver © Ken Rubin), Reinhard Viertl (center), and Michael Oberguggenberger

1988 and became an associate professor in 1992. He was the founding director (1987) of the "Medical Expert Systems Laboratory" at the Department of Medical Computer Sciences, University of Vienna, which became the "Section on Medical Expert and Knowledge-Based Systems" in 1992 and is now part of the Center for Medical Statistics, Informatics, and Intelligent Systems at the Medical University of Vienna.

From 2002 until 2005 he was the founding director of the Ludwig Boltzmann Institute for Expert Systems and Quality Management in Medicine, Vienna, and since 2002 he is a co-founder, CEO and scientific head of *Medexter Healthcare GmbH* (a spin-off of the Medical University of Vienna).

A large part of K.-P. Adlassnig's research is devoted to fuzzy methods in clinical medicine which partly has been described in [18]. The methods he investigated and applied include, on the one hand, fuzzy sets, fuzzy logic, consistency checking in fuzzy systems, fuzzy automata, fuzzy control, and fuzzy programming languages and, on the other hand, computer-assisted diagnosis, medical expert systems, clinical decision support, knowledge-based systems, artificial intelligence in clinical medicine, fuzzy extension of a medical knowledge representation and processing language (Fuzzy Arden Syntax), and fuzzy knowledge acquisition.

The clinical areas of application include *internal medicine* (rheumatology, hepatology, gastroenterology), *laboratory medicine* (toxoplasmosis serology, thyroid hormones), *intensive care medicine* (ICU-acquired infections, respirator control, monitoring of adult respiratory distress syndrome), and *infection control* (healthcare-associated infections).

Especially remarkable are the computer-based medical consultation systems CADIAG (*Computer-Assisted Diagnosis*) whose design and construction was initiated by K.-P. Adlassnig. The aim of these systems is to support diagnostics in the field of internal medicine. CADIAG-1 deals with Boolean rela-

tionships (formulated as if-then rules) between symptoms, signs, laboratory test results and clinical findings, on the one hand, and diseases, on the other hand. Since precise and definite information about real world objects is difficult to obtain and, in the realm of medicine, such information is typically not accessible to physicians when deciding about a patient's diagnosis and treatment, the successor system CADIAG-2 was based on fuzzy set theory.

CADIAG-2 is applied successfully at the Medical University of Vienna, and it has been mentioned in several textbooks [19–21].

The formal logical framework for CADIAG-2 was studied together with Agata Ciabattoni, Thomas Vetterlein and others within the cooperation in the project "Fuzzy Logic: From Mathematics to Medical Applications" which was financed by the Vienna Science and Technology Fund (WWTF). K.-P. Adlassnig is also involved in many other research projects supported by different grant agencies, and in industrial cooperations with partners like VOEST-ALPINE Medizintechnik, HP, PCS, and Siemens.

In the context of Medexter Healthcare GmbH a fuzzy extension of a medical knowledge representation and processing language (Fuzzy Arden Syntax) for representing procedural clinical knowledge in order to facilitate the sharing of computerized health knowledge bases has been developed.

K.-P- Adlassnig and his group received several awards, including the Federal State Prize for excellent research in the area of rheumatology awarded by the Austrian Federal Ministry for Health and Environmental Protection (1987), several awards for research in Medical Informatics and Statistics of the Georg-Grabner-Foundation, University of Vienna Medical School, Austria, and the Homer R. Warner Award at the American Medical Informatics Association Annual Symposium, Chicago (2012). In 2014, he was elected as Fellow of the American College of Medical Informatics (ACMI).

He was involved in the organization of the Tenth International Congress for Medical Informatics in Europe (MIE '91) in Vienna and of the $22^{\text{nd}}$ International Congress of the European Federation for Medical Informatics (MIE 2009) in Sarajevo, as well as in the organization of numerous workshops in medical informatics and conferences on fuzzy logic all over the world. Since 2002 he is editor-in-chief of the international journal "Artificial Intelligence in Medicine" where he was a member of the editorial board since 1988.

The Vienna University of Technology has a long tradition in non-classical logic. Institutions like the Kurt Gödel Society whose list of presidents includes Daniele Mundici (now Università degli Studi di Firenze), Petr Hájek (Czech Academy of Sciences), Gaisi Takeuti (Emeritus of the University of Illinois at Urbana-Champaign) and Dana S. Scott (Emeritus of the Carnegie Mellon University, now Berkeley) are closely related to the logicians and proof theorists at the Vienna University of Technology, in particular to Matthias Baaz, Thomas Eiter, Chris Fermüller, Georg Gottlob, Alexander Leitsch, and Helmut Veith.

One example for this cooperation was the doctoral programme "Mathe-






Figure 6: Agata Ciabattoni at work (left, © Stefan Joham), and a poster of the *Vienna Summer of Logic* 2014

matical Logic in Computer Science" whose partners, the Vienna PhD School of Informatics and the Kurt Gödel Society, joined forces in supervising ten PhD students in the field (2010–2013).

The current doctoral program "Logical Methods in Computer Science" (LogiCS, 2014–2017) is a PhD degree program funded by the Austrian Science Fund (FWF) and run jointly by the three Austrian universities Vienna University of Technology, Graz University of Technology and Johannes Kepler University Linz. Its main topics are databases and artificial intelligence, verification, and computational logic, and its speaker was Helmut Veith. The faculty includes Matthias Baaz, Armin Biere, Roderick Bloem, Agata Ciabattoni, Thomas Eiter, Chris Fermüller, and Alexander Leitsch, there are also guest lecturers like Petr Cintula and Libor Behounek (Czech Academy of Science), and Bruno Buchberger (JKU).

In the summer of 2014, Vienna hosted the largest event in the history of logic. The *Vienna Summer of Logic* (VSL) consisted of twelve large conferences and numerous workshops, attracting over 2000 researchers from all over the world. Attached to the event were, among others, the exhibition "Kurt Gödel and the Origins of Logic in Vienna", and the so-called "Logic Lounge" in the Kunsthalle Wien, featuring Christos Papadimitriou (University of California, Berkeley), Georg Gottlob (Oxford University), Richard Zach (University of Calgary), Roderick Bloem (Graz University of Technology), Byron Cook (University College London), Moshe Vardi (Rice University), Magdalena Ortiz (Vienna University of Technology), and Ruzica Piskac (Yale University).

A particularly active member of the Theory and Logic Group of the Department of Formal Languages at Vienna University of Technology is Agata Ciabattoni. She holds a Master in Computer Science from the Università di Bologna and a PhD in Computer Science from the Università degli Studi di Milano. Prior to her current position, she had a Marie Curie Individual Fellowship



Figure 7: Vienna Summer of Logic 2014 © Philipp Horak for profil

from the European Commission and a Habilitation Scholarship (Elise Richter Programme) from the Austrian Science Fund (FWF). She was a principal investigator of the projects "Semantic Characterization of Cut-Elimination" (2006–2009), funded by the Austrian Science Fund (FWF) and "Fuzzy Logic: from Mathematics to Medical Applications" (2008–2012), funded by the Vienna Science and Technology Fund (WWTF). In 2011 she received a highly prestigious START prize from the Austrian Science Fund (FWF) for her project "Non classical proofs: Theory, Applications and Tools" (December 2011 until November 2017, hosted by the Vienna University of Technology).

In 2007 she co-edited the volume "Algebraic and Proof-Theoretic Aspects of Non-Classical Logics" on the occasion of the $60^{\text{th}}$ birthday of Daniele Mundici [22]. She is a faculty member in the doctoral program "Logical Methods in Computer Science" and played a major role in the Vienna Summer of Logic 2014.

Reinhard Viertl also worked at the Vienna University of Technology in the Department of Statistics and Probability Theory as professor since 1982. He held visiting professorships at the Universität für Bildungswissenschaften Klagenfurt, the University of Innsbruck, the University of Calgary, and the Satya Wacana Christian University of Salatiga, Indonesia and is a honorary member of the Austrian Statistical Society.

He distinguishes different types of uncertainty, namely, variability (stochas-

tic uncertainty), errors (stochastic and systematic), imprecision (data uncertainty, fuzziness), and model uncertainty, and his main interest is the relationship between fuzziness and statistics.

His research topics include Bayesian statistics, the description and analysis of fuzzy data, statistical methods for fuzzy data, fuzzy information and decision making, and time series analysis for fuzzy data, among others.

He published a number of textbooks and monographs along these lines, including [23–27].

At the Viennese universities there were and are several researchers who do or did not work in fuzzy logic proper, but occasionally discussed this topic in some of their courses and seminars. An (incomplete) list includes Robert Trappl and Anton Zeilinger (University of Vienna), Wolfgang Janko and Andreas Geyer-Schulz (Vienna University of Economics and Business), as well as Georg Gottlob, Wolfgang Slany, Gerfried Zeichen, and Franz Ziegler (Vienna University of Technology).

Also Herbert Hörtlehner who taught at a technical high school (HTL Spengergasse), but also at the University of Vienna, deserves to be mentioned.

## 4 Other parts of Austria

Michael Oberguggenberger passed his habilitation at the University of Innsbruck in 1987 and is Professor at the Institute of Basic Sciences in Engineering Science since 2008. He mainly works in the areas of partial differential equations, distributions, stochastic and probabilistic methods and generalized probabilistic methods in engineering and risk analysis, and in the numerics of differential equations. His interest in fuzziness also includes random sets and imprecise probability.

One of the members of his research group, Thomas Fetz, passed his habilitation at the University of Innsbruck with the thesis "Random set models for multivariate uncertainty analysis" in 2011, around ten of the PhD theses he supervised are in the area of fuzziness and imprecise probability.

He was and is involved in several research projects such as "Differential equations with fuzzy parameters" (funded by the Austrian National Bank) and "Computational interdisciplinary modelling", Within the doctoral college DK-CIM, financed by the Austrian Science Fund (FWF).

Industrial partners include Novartis — Biochemie Kundl in the project "Process modelling with fuzzy logic" and INTALES in the project "Innovative COncept for Nonlinear Analysis of Lightweight Structures" (ICONA).

He has co-edited special issues of the *Journal of Applied Mathematics and Mechanics* (ZAMM), the *Journal of Risk and Reliability*, the *International Journal of Approximate Reasoning* and *Fuzzy Sets and Systems* on topics such as imprecise probability, uncertainty in engineering risk and reliability, and differential

equations over fuzzy spaces, and an edited volume on analyzing uncertainty in civil engineering [28].

Presently, Michael Oberguggenberger is president of the Austrian Mathematical Society (ÖMG).

At the Graz University of Technology Helmut F. Schweiger from the research group of numerical geotechnics within the Institute for Soil Mechanics and Foundation Engineering supervised several PHD theses, e.g., "Reliability analyses in geotechnics with the random set finite element method" (Gerd M. Peschl, 2004).

## Acknowledgment

I would like to thank several colleagues for providing me with information for this presentation, in particular Klaus-Peter Adlassnig, Michael Oberguggenberger, and Thomas Vetterlein.

This brief historical overview is also intended to acknowledge the contributions of those friends and colleagues who already passed away: Nicole Blanchard, Dan Butnariu, János Fodor, Siegfried Gottwald, Petr Hájek, Herbert Hörtlehner, George J. Klir, Ladislav J. Kohout, Franco Montagna, Elie Sanchez, Lawrence N. Stout, and Helmut Veith.

R. I. P.

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# Fuzzy systems research in Hungary – a subjective story

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# 1 Fuzzy systems research in Hungary – a subjective story

Although, When I try to write the brief history of fuzzy systems related research in Hungary it will inadvertently become a subjective story, reflecting the view of the author, i.e., myself, and focusing on events and activities where I took part in personally. There is also no chronological order in this article; it is like when memories come to one's mind in a random way, connected by associations…

I read the first fuzzy papers in 1973 when one of the lecturers at Budapest University of Technology (**Miklos Hajnal**) dumped a large amount of journal articles related to various aspects of intelligent systems in front of a group of selected students (in the present Bologna system we were MSc students at the time), with the remark, we should select whichever we found containing interesting topics, and should start research work in the selected field. I do not know why Lotfi Zadeh's famous *Fuzzy sets* article caught my eyes immediately, but it did! And then I took with me all the other papers which had the word "fuzzy" in the title. I still remember Goguen's L-fuzzy sets being among them, and a few others, on fuzzy automata and other motley topics.

I started to read the papers - and a fascinating new world opened in front of me. Since then 43 years passed and this fascination never faded away. What happened to me was like love at the first sight, a love that continued in a happy marriage.

Soon I wrote up my first technical report [1], motivated by an interesting remark I found in a footnote of Zadeh's crucial paper. Although the definitions for union and intersection suggested in the main body of paper were the max and min operators, an alternative pair of definitions what Zadeh called "interactive operators" was what in the modern literature are known as algebraic norms. I found them quite interesting, because I had the intuitive feeling that subjective truth is generally increased by repeated statement, i.e. the strictly monotonic increasing and decreasing property of this pair of operations would reflect more adequately the nature of every day logic than the properties of the standard pair of operations. I found a strong supporting argument in a book by the famous researcher of stress, H. Selye, a scientist of Hungarian descent. This was the book *From Dream To Discovery On Being A Scientist*. In this book Selye clearly states that repeated results of experiments confirming the same hypothesis lead to increasing conviction about the truth of the same. In my report I proposed the argument that interactive operators should be promoted and widely used, and that it was worth while studying their axiomatic properties. When I was

ready I sent a copy of this report to Lotfi Zadeh, to Berkeley.

I was surprised how supportive and positive Lotfi's reaction was! Even though that time I had not met him yet in person, I count his first letter to me as the beginning of a friendship, of course, a friendship between an experienced scientist, a mentor, and a young researcher just starting to open his eyes to the beauty of knowledge and the amazing phenomenon of permanent expansion of scientific knowledge. After his first response Lotfi also sent me a bunch of papers and technical reports from UCB, which I still keep as special treasures in my collection on fuzzy technical literature.

I was very much pleased when I found years later that Lotfi even kept my first technical report in his library and thus later it was included in several bibliographies published in the 1970's and 80's by various authors.

Soon I found out that I was not the only one in Hungary at that time who was somehow interested in the fuzzy world, although I still think today, probably I was the most determined at that time to devote my whole scientific carrier to fuzzy research, and I was the first one to publish international papers.

Although Zadeh published his seminal paper in 1965, in Hungary the first interest in this new area started only in the early 1970-s. **Tibor Vamos**, that time very active in IFAC, knew Lotfi in person and they even became friends. Although Vamos did not publish essentially new papers dealing with fuzzy approaches, in the 1970-s he became a great promoter of fuzzy research in Hungary, and being part of the political establishment and a very influential member of the Hungarian Academy of Science, his support was rather important in a small country governed by leftist doctrines, even in the field of scientific research. Vamos later became the first Hungarian editorial board member of *Fuzzy Sets and Systems*. He also played a crucial role in providing an opportunity for me to meet Lotfi in person in 1975.

In the early 1970-s in several scholarly communities in Hungary the fuzzy idea came up, at least as something worthwhile looking at it. One of these was the research seminar at the Research Institute for Telecommunication, where after some overview seminars by local researchers I was also invited to give several talks in 1974 and 75.

It was in 1975 that I got my MSc equivalent degree in Control Engineering. The topic of the thesis was purely fuzzy. I was also extremely lucky, as in 1974 the Ministry of Education gave permission to the Faculty (School) of Electrical Engineering to introduce an experimental type of doctoral school program, and I was one of the first to be admitted. In this experiment, while working on the MSc thesis the few privileged students were allowed to start a very intensive doctoral training and so it came that

in 1977 I got my PhD (that time called Dr. techn. in Hungary). The topic was, of course, fuzzy control. That way I was the first young researcher in the country who actually earned a doctoral degree by fuzzy research. The topic was so new and unknown that in my degree certificate the field was indicated as "Probability Theory"; meaning that this was the closest "legitimate" field of science. I will come back to the topic in more detail somewhat later.

At the Budapest University of Technology the first who became interested in the fuzzy field was my later friend **Peter Varlaki** who got in contact with fuzzy papers through another friend of his. He remembers those years as follows:

"I could get acquainted deeper with the concept of fuzzy sets (after the cursory knowledge of fuzzy automata) for the first time in 1972 by Gabor Bonyhai. This accomplished scholar of literature and hermeneutics, who died early in 1996, read the original introductory papers of Zadeh on fuzzy sets and thought they could be very important and might have a productive effect on the field of research in general linguistics and the science of literature, as well. In the early 60-s, as a special talented high school student, he started to deal with mathematical logic, the science of modern logic, and with the results of the schools of philosophical logic of the 20-s and 30-s in particular. Thus, he studied *inter alia* the works of Carnap and Lukasiewicz. He dealt with subjective probability approaches closely linked with the fuzzy concept, which played important role in his hermeneutical analyses. He intended to identify and show dynamic systems of value, to be explored at the background of the system of the plot in several major literary works, where during the analysis both the fuzzy and the subjective probabilistic approach played an important role according to his practical investigations. His paper *The Language of Value*, published in German, may be of particular interest, in which the description of value systems was examined using modern linguistic analysis and the theory of communication. In this case, the fuzzy approach could play an important role."

Note that Bonyhai writes in above cited works about "moegliche Welten", i.e. he already recognized the difference between possibility and probability or likelihood. Varlaki further writes:

"Perhaps it is worth to note that already in 1971 I had the opportunity to get acquainted with the idea of fuzzy automata, in particular with regard to the theory of the stochastic and non-deterministic abstract automata. I also talked a lot with my late friend Ferenc Szatmari on the conceptual and philosophical issues of fuzzy, deterministic, non-deterministic and

stochastic abstract automata. Szatmari gave a lecture on the abstract automata and algorithms in 1973 at the First Hungarian Conference on Theory of Systems in Sopron. Later, in the spring of 1977 during my Moscow PhD studies, where, together with my scientific leader Naum S. Raibman, we published subjective probability models using Kalman filters, and I again encountered the fuzzy idea, as Professor Raibmann's good friend and colleague Professor Aiserman was a member of the editorial board of *Fuzzy Sets and Systems*, a periodical just recently started. I remember my trip from Moscow back home, in June, for summer holiday. I traveled on the train together with **Margit Kovacs** and **Istvan Joo**, to whom both I showed the first issue of FSS. I suggested Margit, who specialized in the Tykhonov regularization method in connection with numerical analytic problems, that she should study fuzzy theory, since it could obviously mean a compensation of regularization type for solving tasks in connection with ill-posed problems. Much later Kovacs started to deal with fuzzy sets actively, and she became the author of several papers in this field. Also, I wrote several papers together with her, especially on the modeling of specific economic and econometric problems. This way I inadvertently contributed to starting the opportunity that Kovacs's excellent students, **Janos Fodor** [2, 3] and **Robert Fuller** [4, 5, 6], who became really successful in the fuzzy field.

From the 90's, together with my colleague and friend Laszlo T. Koczy, and with Istvan Joo, a more than twenty-fold co-author of Pal Erdos, we have elaborated important issues on fuzzy interpolation. In the late 1990s and early 2000s, I had the pleasure of carrying out research work with **Peter Baranyi**, a former student of Laszlo, addressing special fuzzy linguistic problems.

In around 2010, I joined my above mentioned friend Robert Fuller in his research work, which he had carried out for years in the field of possibility correlation theory. Together with my student **Istvan Harmati**, and Robert, we constructed and tested new possibility measures of dependence [7], primarily on the basis of Alfred Renyi's approximation method of stochastic dependence measures. Recently, together with Harmati and my other friend **Laszlo Szeidl**, we attempted developing the theory of fuzzy number stationary time series."

**Peter Foldesi** who is at present the Rector (President) of Szechenyi Istvan University was also among the PhD students of Varlaki. In the 2000-s he started to work with me and we published jointly some fuzzy papers related to basic problems of logistics. An example is the Traveling Salesman Problem with fuzzy constraints and weights [8].

After citing Varlaki remembering his own fuzzy story, we may come back to the mid 1970s. One of the most active and earliest Hungarian fuzzy pioneers was **Jozsef Dombi**. He started to work with Hans-Jurgen Zimmermann, and spent several years with him in Aachen. The most important contribution of Dombi to fuzzy science is undoubtedly the pair of fuzzy operations (intersection and union), which are known and cited in the literature as Dombi-operations (cf. [9]). Throughout the decades Dombi did a lot of work on fuzzy aggregations (e.g. [10]) and in 2011 he succeeded in obtaining the highest Hungarian earnable scientific degree: DSc from the Hungarian Academy of Science, which is a postdoctoral title rather hard to get, and what is considered to be much more prestigious than a full professor's title (which is also conferred centrally in Hungary, by the government, although a condition of initial conferment of the title is the recommendation of a respective university). I was one of Jozsef's reviewers in the procedure, as by then I already had had that title for a while – as first in Hungary in the fuzzy field.

Personaélély for me it was quite a surprise when we recognized some much unexpected properties of the Dombi operations when we worked on artificial neural network construction with my former PhD student **Rita Lovassy**, starting from fuzzy operations and fuzzy flip-flop units (which I will mention later) [11]. We compared all available fuzzy aggregation families (later, by the idea another PhD student of mine, **Laszlo Gal**, we even proposed a new aggregation pair [12]), and we found that the learning properties derived from all these fuzzy operations were best for Dombi's ones! Thus we presented the results in our publications and it is no surprise that Jozsef was quite happy to learn about our results. It is worth mentioning that the best networks we thus constructed with Rita had several advantageous properties compared to e.g. the widely used Mathlab NN Toolbox. Our fuzzy neural network lacks the disadvantageous overfitting property and provides a smoother and more realistic fit, even in the presence of outliers.

With Jozsef we often debated about which of us to was the first active fuzzy researcher in Hungary. Anyway, we kept our friendship since the 1970-s, and even nowadays we occasionally invite each other to the committees of our conferences, or PhD exams. It is worthwhile mentioning about Jozsef that – despite being a mathematician – he founded his private company for consultancy and IT solutions, among others offering practical help in large size logistic optimization tasks. Fuzzy can help in so many different fields!

Returning to the historical decades, in the 1980-s I prepared my CSc

thesis (a kind of elevated PhD type degree, abolished in 1992, introduced originally in the Soviet Union, but taken over by most Eastern European countries, that was issued centrally by the respective national Academies of Science), and submitted it to the Hungarian Academy of Science. The topic was disguised as the optimization of hierarchical structured telecommunication networks, because in those days fuzzy research was rather frowned upon by official Hungarian research gurus. The real topic was however the optimization of fuzzy trees and the application problem could have been the optimization of hierarchical networks of human agents, management structures or networks providing services or material flow with varying availability or throughput [13]. In 1989 I got the degree and thus became the first CSc in fuzzy in Hungary!

Rather in parallel with my efforts to obtain official recognition to fuzzy research by the omnipotent Academy, Margit Kovacs also worked on a fuzzy topic at Eotvos Lorand University: fuzzy mathematical programming [14]. She received her CSc degree in 1990, and even though her main topic was not fuzzy programming, she had mentioned fuzzy options in her thesis. It is a pity that later she turned her back to fuzzy research, even denounced fuzzy as a "useless field". In 1990 however, Margit was still very enthusiastic and proposed the foundation of a Hungarian Fuzzy Association. Some 30 researchers came together and elected me to President of the Association. (In fact, to Co-president as I proposed on site that we elect Tibor Vamos in his absence to another Co-president, due to his continuous strong support of the field.) Margit was appointed Secretary General of HFA.

At this point a very important event connected to Vamos must be mentioned. Yet in 1975 he organized a bilateral conference at his institute, the US-Hungarian Joint Seminar on Pattern Recognition. Among the invited speakers from the American site was Lotfi Zadeh, the famous Father of Fuzzy Sets! Each foreign guest speaker was assigned a Hungarian "lance-bearer" by Vamos, and in Lotfi's case I had the honor. This was an excellent opportunity to spend a short



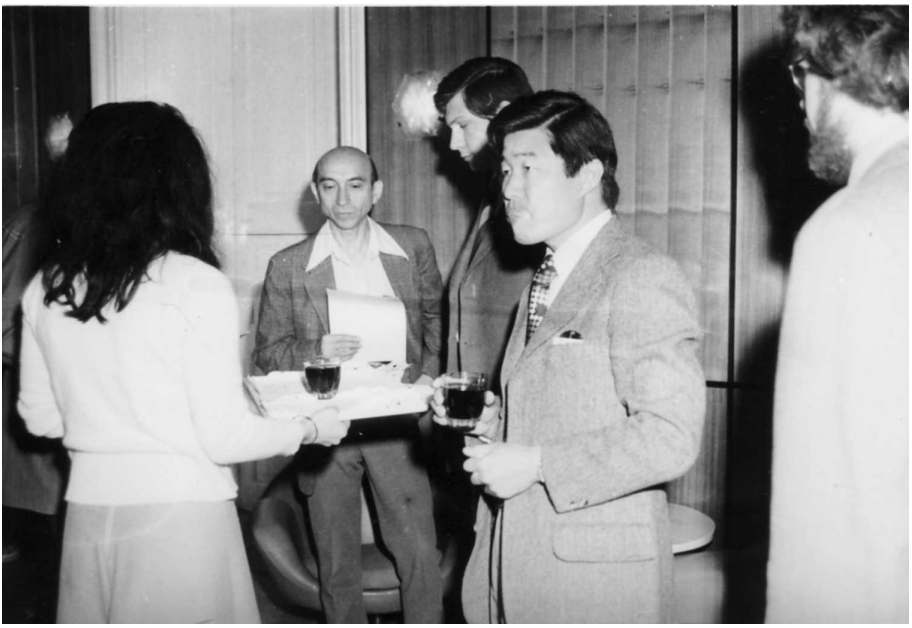
*One of my first discussions with Lotfi in 1975 in Hungary*

week with Lotfi and his wife Fay. I remember that I was amazed how modest, almost puritanical this famous researcher was. During those days I learned a lot from him about how to proceed with my fuzzy studies. The Seminar itself offered me a lot of new knowledge, in many respects. I was surprised, because all invited Hungarian speakers discussed one or another aspect of statistical pattern recognition, while the guests covered a very wide scale of different approaches, including semantic and, of course, fuzzy pattern recognition techniques. I realized then and there that Hungarian science had a certain "benevolent dictatorship", where some influential scientists always unambiguously determine the "only feasible" approach to a given problem. In a way it is still like that in my country...

Another important moral of the conference for me was the reaction to the fact that I received an offer for a PhD scholarship at the University of Pennsylvania, with a joint MSc scholarship to my freshly married wife studying control engineering at the time, by an another American participant: King-Sun Fu. When I namely mentioned about this offer to my university superiors the answer was "There are many other more senior colleagues waiting for such an offer. You must be patient until your turn comes!" Of course these "senior colleagues" had been members of the Communist Party, and had had no research activity at all...

So, I had to do my (first) doctoral degree in Hungary, as I mentioned it

earlier. The topic I chose was the axiomatic system of a fuzzy algebra of which algebraic operations and more general Hamacher-operations were a possible representation. Based on this, a new fuzzy control technology was proposed (very different from Mamdani-control, an extension of the idea of bang-bang control), and it worked rather well for some simulated systems. In that thesis I proposed – as first in the literature – inverse set operations for strictly monotonic norms and co-norms; and based on those a successfully applied simple clustering algorithm was also presented. This research resulted in my first international journal papers, among others my first paper in Information Sciences.



*Coffee break of the conference, I am talking with Lotfi; in the foreground Kenji Hanakata*

1975 also brought some other important events for me. In the summer I attended two international conferences, one in Edinburgh (presenting the fuzzy clustering method applied for karyometric classification), and one in Bucharest, the congress of the World Organization of General Systems and Cybernetics (where I presented my first results on strictly monotonic norms and algebraic systems) [15]. At this conference I met Hans-Jurgen Zimmermann and participated with him in the formation of the EURO Working Group on Fuzzy Sets.

After 1975 however, for a very long time I had no chance for participation at conferences in the Western World, because in 1976 I was

allegedly involved in "activities against the state". Nevertheless, I continued working and sent some manuscripts to international journals. I also tried to stay in connection with some of the researchers I met in 1975 at one of the two international conferences, but my letters and reprints I sent to them often returned with the remark "addressee unknown" (even though I sent those letters to the addresses on the business cards I had from them). Luckily this was not the case with Lotfi, who continued sending me papers and reports, and also not with leading Japanese fuzzy researcher Toshiro Terano whom I also exchanged letters and papers with during the next years. I only had a chance to meet Prof. Terano in person much later, in the late 1980-s, and after that I also had the privilege to call him another fatherly friend.

Nevertheless, in this period of relative isolation I still could attend several fuzzy events in the Eastern half of Europe, among others the famous fuzzy meeting in Eisenach, in the Wartburg castle – where I met the second time my later close friend Peter Klement. One year earlier he had sent me an invitation to give a couple of talks at Johannes Kepler University in Linz (Austria), where I could present my ideas about monotonic norms, inverse operators, etc. Although Peter sponsored my trip completely I was allowed by the Hungarian authorities to stay on site only three days, anyway, during those three days I could suck in an incredible amount of information from Peter's fuzzy library.

Using a bilateral exchange trip later I also could visit Hans in Aachen, where I presented a seminar talk on a new fuzzy similarity degree. Later I attended all fuzzy conferences of Hans in Aachen with talks – but that happened already in a completely different political situation...

In 1987 my personal position suddenly improved as the political system was somewhat relaxing and applying to an American foundation I got support for participation in the 3rd IFSA World Congress in Tokyo where I presented some results on describing the relative position of two components of an image by a fuzzy degree [16]. This research lead soon to a journal paper [17].

This was the first IFSA WC that I could attend, and ever since I never missed one... In Tokyo I met Kaoru Hirota with whom we published afterwards a large number of papers, first on Fuzzy Flip-Flops [18], later on Fuzzy Rule Interpolation [19, 20, 21], a topic that I still now consider as my most important contribution to the fuzzy field (and which topic also brought a large number of citations by other researchers). From that year I could annually participate in some major fuzzy conference and I started vivid personal interactions with many fuzzy scientists around the world. In

1988 I went to Iizuka, and attended later all subsequent Iizuka conferences. I attended in 1989 the IFSA WC in Seattle, and have attended all other IFSA WC-s since then. In addition in 1988 I also could go to Egypt, to a conference that later became notorious among the many Japanese participants, because of many organizational mistakes – nevertheless we met there with Kaoru and had a chance to discuss further collaboration.




*At the IFSA WC in Tokyo, with Kaoru Hirota. On the left, Witold Pedrycz, on the right the late Ernest Czogala*




*At the IFSA WC in Seattle (1989) with Christer Carlsson, Janusz Kacprzyk and Kaoru*

1990 brought at last democracy and freedom to Hungary and thus establishing societies and associations became possible – this is how the Hungarian Fuzzy Association was founded. Among the initial members there were also Margit Kovacs's two excellent PhD students: Janos Fodor and Robert Fuller.

Janos got his CSc degree soon after this founding event, in 1991, with a thesis on fuzzy preference modeling. Ever after he continued research in this field and got his Dr.habil degree (one more exotic postdoctoral title in the complicated labyrinth of the Hungarian system of scientific degrees and titles!) in 2000 and the DSc title in 2004 with the thesis "Preference structures and aggregation procedures". In all these degree defense procedures I was given a role as an examiner or committee member.

Janos collaborated with many famous fuzzy researchers, such as Marc Roubens, Bernard De Baets, Radko Mesiar, Didier Dubois and Henri Prade, Ulrich Bodenhofer, Ronald Yager and probably he became the most cited Hungarian fuzzy researcher. I helped him all along his career, supporting some of his first trips from my own grants, and by recommendation letters. We also had a joint paper with Janos when in 1991 Kaoru and I organized

the First Joint Hungarian-Japanese Symposium on Fuzzy Systems and Applications, with Tibor Vamos and Toshiro Terano as Honorary Chairs. Later we organized a series of international fuzzy conferences in Budapest jointly with Janos, having such prominent plenary speakers as Lotfi and Hans among many others. Janos passed away in 2016 as Rector of Obuda University – a grave loss to the Hungarian and the international fuzzy community!




*At the Sphynx with Kaoru – the beginnings of fuzzy rule interpolation*

One of the PhD students of Janos deserves special attention: this is **Sandor Jenei**, a very talented mathematician who achieved results in the construction and structural description of residuated monoids, the standard completeness, and monoidal t-norm logic, which results had a strong impact on fuzzy algebraic structures. The latter is a natural continuation of similar results on Lukasiewicz and Godel logics. Further he has results on the geometry of associativity, in connection with full Lambek algebra with exchange (FLe-algebra). Sandor has a large number of publications, some jointly with B. De Baets or F. Montagna [22, 23].



*As General Chairs of the First Joint Hungarian-Japanese Symposium on Fuzzy Systems and Applications, with the two organizing secretaries*

The other excellent member of the Eotvos Lorand University group was Robert Fuller who started fuzzy research even earlier, receiving the first doctorate in 1984 for his thesis on Fuzzy Spaces and their Fixed Point Theorems. In 1987 he obtained CSc from Moscow State University in Fuzzy Linear Problems. In 1999 he got Dr. habil. for fuzzy optimization, and eventually in 2015 the DSc title for Multicriteria Decision Models with Imprecise Information. In the last two procedures I participated again as committee member and examiner.

Robert also spent a period with Hans Zimmermann in Aachen, and then he moved to Turku in Finland to the Abo Akademi University, collaborating first with Patrik Eklund, later with Christer Carlsson, as a Finland Distinguished Professor. In the meantime he also visited the University of Trento in Italy collaborating with Mario Fedrizzi. Robert's wide spread research activities cover fuzzy probability, OWA operators, neuro-fuzzy systems and a large number of real life management and economics applications of fuzzy models and algorithms.

Continuing my personal story in the fuzzy context, in 1990 I received an invitation from James Keller to the University of Missouri in Columbia as

a visiting professor. It was a grimace of fate that I had not been granted the necessary working visa by the USA, because I was coming from a recently communist country. In all my active life until 1990 I was discriminated, even persecuted because of my conservative political views by the dictatorial state in Hungary, and in 1990 I was rejected by the West because of my Hungarian citizenship! And, of course, because of by then fuzzy was a very sensitive research field, which had been recognized as crucially important for the defense and space industries.

The USA being closed for me I was looking for another position abroad, and in 1992 I went to Korea, as visiting professor at the Pohang Institute of Science Technology, an excellent private university – later appearing in some university rankings as Nr.1 in the world among universities not older than 50 years. In Pohang I developed a Fuzzy Systems course using George Klir's excellent textbook and my own research.

From there I went soon to Japan and in 1993/94 I held the Fuzzy Theory Chair at the Tokyo Institute and Technology – a very honoring and prestigious position that I definitely thanked to my successful collaboration with Kaoru Hirota and to cordial relations with Toshiro Terano. In Tokyo I further developed the Fuzzy Systems course and taught it to a large class of graduate students, many of them being students of Michio Sugeno. During this time I wrote several papers on Fuzzy Rule Interpolation (FRI, see earlier) with Kaoru and one journal paper on the explicit mathematical expressions of fuzzy controllers with Michio [24], and a few more papers with many Japanese and other co-authors.

The Fuzzy Systems course was further developed later and it has been taught in Hungary (in Budapest, where it is an elective course now, and in Gyor where it was included in four new master programs, as an obligatory). I also delivered that course in several other European countries, such as in Austria, Italy and Poland; and at several Australian universities. My textbook on fuzzy systems in Hungarian language (with two editions as far) was also based on this course [30].

Besides this course, in some years when there was enough interest I have been teaching another elective on more advanced topics of fuzzy systems (FS 2). I also have had a series of MSc and PhD students doing fuzzy research. The first programs for Fuzzy Rule Interpolation simulation and some new technics of the FRI algorithm family were written by my MSc students **Akos Juhasz** and **Gyorgy Vass**, and others, in the late 1980-s and early 1990-s.

My first purely fuzzy PhD student**, Szilveszter Kovacs** did a thorough analysis of the non-linearity of membership degrees obtained by linear rule

interpolation, establishing conditions for acceptable approximations [25]. On the other hand, he proposed a new type of FRI algorithm based on Kruse's and Klawonn's vague environment. This latter was our first successful real application, the control of an automatic guided vehicle, where the size of the fuzzy control rule base could be drastically reduced by FRI, while keeping all advantages of the original fuzzy control. Szilveszter is now an Associate Professor of the University of Miskolc and Vice-President of the Hungarian Fuzzy Association.

From the mid 1990-s I started to be intensively interested in automatic rule based generation based on input-output data, motivated by the work of Michio (with his student T. Yasukawa). With Alex Chong, whose supervisor Tom D. Gedeon asked me to join him in the supervisory panel at Murdoch University in Perth; we achieved quite good results in solving a difficult modeling problem, the evaluation of geological data from the point of view of the likelihood of finding petroleum in an area (estimation of the building porosity). Identifying the hierarchical fuzzy interpolative rule based model by applying fuzzy clustering led to a better fitting model than any previous one [26].

Since 1994 we have worked closely together with Tom, among others, proposing new FRI approaches and later also in the area of Fuzzy Signatures (FSig), often co-supervising postdocs, PhD and master students, while I was partner or co-investigator of quite a few ARC grants. In the past two decades I spent about two and a half years at various Australian universities doing joint research with close friend Tom and other colleagues. I want to mention Kevin Wong, who joined Tom at Murdoch as a postdoc, and with whom we did quite a lot of joint work, mainly in applying Fuzzy Signatures, which collaboration continued when Kevin left for Nanyang Technical University in Singapore, and also when he returned to Murdoch while Tom was already at the Australian National University. At ANU our joint PhD student was Sumudu Mendis, with whom we achieved quite nice results similarly in the Fuzzy Signature field [27]. When I was teaching the fuzzy systems course at ANU in 2015, Sumudu led the tutorials for my lectures. Many of my Hungarian PhD students got involved in collaboration and joint publication with both Kevin and Sumudu, when I arranged visiting stays for them in Tom's research group.



*Kaoru in Budapest, at one of his frequent visits to Hungary with my student Peter Baranyi, who became my son-in-law in the next year*

Among a long series of Hungarian graduate students, I must mention several, who did remarkable work in the fuzzy field. **Domonkos Tikk** with a degree in mathematics went into the rather ambiguous field of fuzzy systems as universal approximators. While quite a few papers were published in this respect from the early 1990-s by various researchers, none of the authors went into the computational complexity aspect of the matter. When Peter Klement with his PhD student Bernhard Moser and other colleagues published an extreme result, proving that virtually any function can be generated by only two fuzzy Takagi-Sugeno rules, it became obvious that these results were rather relative, as the difficulty was pushed from the system to be modeled into the creation of the membership functions themselves, thus involving comparable difficulties and resource intensity. After we started to closely collaborate again (myself teaching Peter's fuzzy course in two academic years in Linz) I suggested Bernhard to go along with the complexity issue, and in his later thesis he presented a very important theorem on the nowhere denseness of the functions created by bounded size Takagi-Sugeno controllers [28]. My Hungarian student Domonkos continued this research and proved a similar theorem for the Takagi-Sugeno-Kang

controller class as well [29]. It was Domonkos who co-authored my fuzzy textbook I mentioned earlier that is still in use at several universities in Hungary [30].

**Peter Baranyi** joined me with bringing a brilliant new idea in the FRI area. He proposed a new algorithm extending the original FRI family to rule bases with subnormal and non-convex membership functions, thus virtually to any kind of rule base [31]. I think Peter was my most talented PhD student ever. He is very active these days as a professor and researcher in several fields from virtual reality to tensor product transformation, that latter including also Takagi-Sugeno controller related issues. It is an interesting fact that he also became my son-in-law and father of my two eldest grandchildren, these days two sweet teenager girls.

In the late 1990-s I continued the works started with Alex Chong, however applying evolutionary and memetic algorithms for rule base identification. An idea coming from the neural network field, the application of the Levenberg- Marquardt algorithm proposed by my good friend and collaborator Antonio B. Ruano, that I tried to combine with Takeshi Furuhashi's Bacterial Evolutionary Algorithm produced a very efficient model identification method. Several graduate students were involved in this research, the most advanced algorithms proposed by former PhD student Laszlo Gal, whom I mentioned earlier.

Another family of local search methods, applicable to discrete problems (n-opt, n=1,2,3; and bounded n-opt) led to very competitive results in optimizing NP-hard problems, especially the Travelling Salesman Problem, producing in some reference cases better results than the standard Concorde approach. The best results in this direction have been achieved with my current student **Boldizsar Tuu-Szabo**.

One more former student, **Krisztian Balazs** must be definitely mentioned as with Krisztian we did a rather thorough comparison of various evolutionary and memetic algorithms, based on a large set of reference data sets, and applying them for the construction of fuzzy rule based models. These extensive comparisons provided firm evidence that the Bacterial Memetic Algorithm I had proposed earlier is the best or at least one of the best available population based meta-heuristics. Applying this approach to the identification of hierarchical interpolative rule based models brought us a Best Paper Award from FUZZ-IEEE in 2011 [32, 33].




*With my PhD students Krisztian Balazs, Rita Lovassy, Laszlo Gal at a workshop in Prague*

Besides the methodological research we have done in this past decade in considerable amount of application oriented, or at least application triggered investigation was carried out, mostly with colleagues, PhD students and postdocs.

The application of Fuzzy Signatures and its extensions for mobile robot collaboration was started with Tom, and continued with PhD student **Aron Ballagi**, along with my friend **Claudiu R. Pozna** [34]. The extended idea of Fuzzy Signatures, leading to spatial Fuzzy Situational Maps (FSM) was used beside robot communication and collaboration, also for modeling 3D warehouses from a logistics optimization point of view.

As I have mentioned it earlier, we researched other logistics problems jointly with Peter Foldesi. These have included the TSP problem already mentioned, a fuzzy approach to bin packing optimization and various network and resource allocation optimization problems.

FSig could be also efficiently used for the evaluation and optimization of assessment of the condition and optimization of of residential buildings. Several results led to two PhD degrees (**Adam Bukovics** and **Gergely**

**Molnarka**), and investigations of the sensitivity of such evaluations for various aggregations used in the FSig trees (with Istvan Harmati) produced also interesting results.

Another interesting line of application, modeling environmental management systems (proposed by former PhD student, **Adrienn Buruzs**) triggered our research activities concerning Fuzzy Cognitive Maps [35]. Some quite new algorithms for refining and reducing FCM models have been intensively researched with **Miklos Hatwagner**, later in an international collaboration context (with Elpiniki Papageorgiou, Engin Yesil, Vesa Niskanen and others) [36].

I never gave up image processing and recognition type research, and **Alex Tormasi** is just now in the procedure of submitting his thesis on single and multiple stroke handwritten character recognition where fuzzy technics were successfully combined with evolutionary meta-heuristics.

Telecommunication network management and optimization type problems were investigated with several graduate students, one of them being my other PhD student who just submitted his thesis, **Ferenc Lilik**, Here we combined fuzzy rules with wavelet transform, collaborating with **Szilvia Nagy**, and as a result, the necessary measurement quantity for good pre-assessment of the band width of telecomm wire pairs could be essentially reduced.

We also ventured in the area of community networks, first with Alex Tormasi and a group of other students, then, with one of my most junior students, **Gergely Posfai**, where recommendation systems could be improved by applying fuzzy elements.

The long line of students, postdocs and collaborators could be continued, but at the end of this subjective story I intend to deal with our involvement in the international fuzzy community and its institutions.

As I mentioned it above, in 1975 I joined the EURO WGFS. Later it was substituted by EUSFLAT, which is still the major European organization for fuzzy researchers. Although I was there when the idea of founding EUSFLAT was coined, later it came to some disputes: Should the HFA remain independent while EUSFLAT is already in existence? The Hungarian fuzzy community decided for yes.




*At the IFSA WC in Istanbul in 2003, at the end of my presidency. Dan Ralescu, Bernadette Bouchon-Meunier and Kaoru Hirota are also in the picture.*

Soon after the EURO WGFS IFSA was founded, and I got invitations to the IFSA congresses - unfortunately I was not allowed to go. It was only in 1987 that I could attend the 3rd WC in Tokyo, but ever since I was present at every IFSA congress. Already in the early 1990-s I became very much interested in IFSA and in 1995 I was elected a Vice President. This was the time of transitions as IFSA changed from a society of individuals into an association of societies. I was involved in the formulation of the new bylaws, and later in the establishment of various IFSA awards. The logo of IFSA was designed by me and a long gone Hungarian artist who also made the IFSA Award plaque in bronze. Later, Tom Gedeon and I designed colors to the logo while I stayed at the UNSW in Sydney. This logo is still in use in an unaltered form.

After four years of service as VP I was elected President Elect in 1999. When in 2001 I took over the President's position, dark clouds gathered over IFSA. The organization had never been registered officially in any country of the world, the account always moved with the Treasurer from country to country... and suddenly we faced anti-terrorism laws, which prevented the transfer of money without a good explanation. NAFIPS and some other member organizations insisted on the registration of IFSA as a non-profit

organization – while SOFT and a few others declared, they would leave IFSA if it became an official association, because of the additional costs. I was very proud when I proposed the compromise solution that is still working nowadays: having two IFSAs, one registered, and one keeping the original loose structure. In 2003 I could hand over presidency with the feeling of having done a good job!

When I finished my 10 years' service in IFSA I suggested my good friend **Imre Rudas** to be elected as Treasurer. Imre has not yet been mentioned in this history, but in fact he wrote a lot of papers on fuzzy operations, robotic applications, and many other topics in the field [37]. He had also co-authored papers with almost all Hungarian fuzzy researchers, having a very collaborative personality. So, we also had a few papers together, especially with my former student Rita Lovassy, where various fuzzy operations were investigated from the point of view of how to build the best neural networks from them. Imre made the Hungarian Fuzzy Association very active, and started a series of annual conferences in Hungary and neighboring countries with a Hungarian minority, thus involving many scientists who had had little opportunity without those events to appear at an international event. Imre became Rector of Obuda University and made it to a fuzzy and intelligent systems research and events hub.

At last I want to mention that the Hungarian fuzzy community has been also rather active in the IEEE. At first the SMC Society, later the NN Council and its successor, the CIS attracted many of us. In 1992 the first FUZZ-IEEE was held in San Diego, and I attended it, traveling from Korea to the USA. Janos Fodor was also there. Ever after, I have attended all FUZZ-IEEE conferences, also the ones in the frame of WCCI-s... Janos stopped traveling much, so later often I was the only Hungarian participant, sometimes however attending with my students. It was however not so in 1998 when in Anchorage we had a delegation of four Hungarian participants, apart from me, my sister **Annamaria Koczy Varkonyi**, who had also published quite a few fuzzy related papers [38]. Peter Varlaki and our one time co-author, Antal Penninger attended the meeting.

In 2004 I even organized one FUZZ-IEEE in Budapest, with quite a success. In the next year I was elected AdCom member of CIS, and after two periods, I was delegated to the Systems Council of IEEE as the representative of CIS for two subsequent years. Imre Rudas has been active in the Industrial Electronics Society until recently, being a member of the Board of Directors, and holding other positions.

We are still organizing our respective conferences mostly annually. 9

years ago I started the annual Gyor Symposium on Computational intelligence that evolved in the meantime into the European Symposium on Computational Intelligence and Mathematics (ESCIM), jointly chaired by Jesus Medina from Cadiz and Janusz Kacprzyk from Warsaw, having a strong focus on fuzzy mathematics and applications. Imre annually hosts the CINTI (IEEE International Symposium on Computational Intelligence and Informatics) meetings in Budapest, which this year had its 17th issue. I must mention a personal remark: When I reached 60, a special one day Symposium within CINTI was dedicated to my own and my group's work on fuzzy, with a special separate volume of the proceedings – a really kind present from old friend Imre Rudas!



*At the FUZZ-IEEE congress in Anchorage (1998) with the Hungarian participants surrounding Lotfi: Peter Varlaki, my sister Annamaria Koczy Varkonyi, myself and Antal Penninger*

We are still active, we are working on many aspects of fuzzy science, mathematical and theoretical, modeling and algorithmic issues, and many applications from engineering to decision support, from management to legal modeling – but gradually a new generation takes over, and it is fine so!

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# A Brief History of Fuzzy Logic in the Czech Republic and the Significance of P. Hájek for Its Development

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**Abstract** In this paper, we will briefly look at the history of mathematical fuzzy logic in Czechoslovakia (and the Czech Republic) starting from the 1970s and extending until 2009. The role of P. Hájek in the development of fuzzy logic is especially emphasized.

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## 1 Introduction

This paper presents a brief look at the history of mathematical fuzzy logic in Czechoslovakia (and the Czech Republic) from the 1970s to 2009. We will especially emphasize the crucial role of P. Hájek in its development. Let us recall that in 1993, Czechoslovakia split into two countries: the Czech Republic and the Slovak Republic. Therefore, we will refer mostly to mathematical fuzzy logic in the Czech Republic.

This paper is founded on personal remembrances, and claims neither completeness nor objectivity. The list of publications presented accompanies our description and is far from being complete.

We focus on mathematical fuzzy logic and some closely related topics. Therefore, various other branches intensively studied in fuzzy set theory as well as in logic are not included.

It should be noted that some areas that we suppose to be less known are discussed in more detail. Finally, let us remark that occasionally we accompany the description of our memories with mathematical concepts or formulas to provide a more clear or specific explanation but that, because of different focus of this paper, we cannot be too detailed or exact.

## 2 The Seventies

There was no significant movement in life in Czechoslovakia in the seventies. Those years were negatively influenced by communist normalization after the year 1968, and so those who stayed in Czechoslovakia[1] were subject to various kinds of restrictions motivated by the communist party's attempt to have a voice in everything. This pressure was especially serious in the humanities. Luckily, in mathematics, the pressure was not as strong because orthodox enthusiastic communists were not scientists and so they could not understand abstract sciences such as mathematics. On the other hand, access to western literature was quite limited, mainly for economical reasons rather than political ones; Czechoslovakia simply had very limited sources of western currency, and thus, research was partly closed off from western science. Luckily, a lot of good western books were available in Russian because the Soviet Union at that time did not respect copyright laws and freely published most good English books in Russian.

With regard to fuzzy logic, two great ideas were raised in these gray days. The first idea was the concept of *Alternative Set Theory* (AST), and the second idea was establishing a precise formal system of fuzzy logic that is now known as *fuzzy logic with evaluated syntax*.

The basic concept in AST, with a motivation similar to that of the fuzzy set, is that of *semiset*. This is a proper class (not a set) $X$ as a subclass of some set $a$, i.e.

$$X \subseteq a. \tag{1}$$

[1] Many thousands of mostly well educated people emigrated from Czechoslovakia in 1968-69.

As has been shown by P. Vopěnka and P. Hájek in the book [72], semisets can be consistently introduced inside classical Gödel-Bernays axiomatic set theory by slightly weakening its axioms. Note that semisets are by no means as strange as they might seem at first glance. For example, let us consider a class *Sm* of *small natural numbers*. Such numbers are 0, 1, ... but this sequence is not finished — there is no last small number and, therefore, also no first large number. On the other hand, there is surely a number $n$ that is large (i.e., surely not small); for example, $n = 10^{10}$. Thus, we have $Sm \subseteq n$. Let us stress that *Sm* cannot be a set because then it would have to be finite (because $n$ is finite) and so the last small number would have to exist. The class *Sm* is a typical semiset.

These considerations led P. Vopěnka to develop the AST theory. In about 1974, a group of logicians[2] had gathered around him and commenced developing a new set theory that it was thought might become an alternative (hence the name) to the classical Cantorian one (ST). Several important principles of AST depart from ST. The most important among them is a new model of infinity. In classical set theory, an infinite set is obtained via its never-ending extension by new elements, while in AST, the infinity emerges when one is dealing with sufficiently large but otherwise classically finite sets. We arrive at the concept of the so-called *natural infinity*, which, unlike ST, is directed inside: a set is infinite if it contains a proper subclass. In other words, a part of an infinite set is not a set. One can verify that $a$ in (1), as well as the number $10^{10}$ in our example, is infinite. The crucial concept in AST is that of a semiset $\mathbb{FN}$ of all *finite natural numbers* that is a proper subclass of a class $\mathbb{N}$ of all natural numbers. This means that any number $\alpha \in \mathbb{N} - \mathbb{FN}$ is infinite. But recall again that such a number $\alpha$ is *classically finite* (though very large in the given context). We encountered this infinity phenomenon emerging inside very large sets. The semiset $\mathbb{FN}$ is *infinitely countable*, while $\mathbb{N}$ is *uncountable*.

In addition to the many papers published, mostly in the journal CMUC (Commentationes Mathematicae Universitatis Carolinae), the small book [73], written by P. Vopěnka, was published. It presents the essential results for AST up to 1979. Ten years later, a much more extensive book [74] presenting a wide array of well established philosophical background material for AST appeared. Unfortunately, it has never been translated into English. The most recent book on AST focusing especially on its philosophy is [75].

The reader has surely noticed that elements of the above-introduced semiset *Sm* represent numbers of grains that do not form a heap in the known *sorites paradox*:

> *One grain does not form a heap. Adding one grain to what is not yet a heap does not make a heap. Consequently, there are no heaps.*

This paradox is resolved in AST. Moreover, semisets represent vaguely defined classes: i.e., just the objects that are in fuzzy logic modeled by *fuzzy sets*.

The *fuzzy logic with evaluated syntax* $\mathrm{Ev}_{Ł}$ was developed by J. Pavelka in his PhD thesis defended in 1976 and published as a sequence of three papers in [63]. Let us

[2] Among other active people in this group, let us mention that K. Čuda, A. Sochor and J. Mlček.

emphasize that the main reviewer of the thesis was P. Hájek. This was probably his first encounter with mathematical fuzzy logic.

Pavelka's papers contain full descriptions of the formal system of propositional $\mathrm{Ev}_Ł$ including the algebraic analysis of possible structures of its truth values and its metatheory. The third of the papers [63] contains a complicated algebraic proof of the syntactico-semantical completeness of $\mathrm{Ev}_Ł$.

The second of the papers [63] contains a deep algebraic analysis of the possible structures of truth values and refers especially to the concept of residuated lattice, thus anticipating the theory of fuzzy logic developed later by P. Hájek. The basic algebra considered there is (what is now called) *standard Łukasiewicz MV-algebra*

$$\langle [0,1], \vee, \wedge, \otimes, \rightarrow, 0, 1 \rangle \tag{2}$$

where $\otimes$ is the Łukasiewicz conjunction (also called the Łukasiewicz or bold product) defined by $a \otimes b = \max(0, a+b-1)$ and $\rightarrow$ is the Łukasiewicz implication defined by $a \rightarrow b = \min(1, 1-a+b)$, $a, b \in [0,1]$. Another structure considered is the finite Łukasiewicz chain (Pavalka proved that the infinitely countable chain is not acceptable).

The logic developed by J. Pavelka is consistent with the principal idea of fuzziness, namely not only by assuming truth values between 0 and 1 but also allowing axioms not to be fully convincing. This means that a formula $A$ can be an axiom only in some (arbitrary) degree[3]. This assumption implies the concept of proof accompanied by a value that expresses the degree to which the proof is convincing. As a result, we arrive at the concept of *provability degree* (of a formula $A$). The prevailing principle in $\mathrm{Ev}_Ł$ is the *maximality principle* stating that if there are more proofs of a formula $A$ with various degrees then the final provability degree is equal to the maximum (supremum) of all of the former ones. This also implies that all these degrees are equally important; essentially, if we cannot reach 1, then any other degree is satisfactory, provided that we know that it cannot be increased.

A very important consequence of metamathematical considerations about $\mathrm{Ev}_Ł$ is the theorem stating that, if we consider a residuated lattice on $[0,1]$ and the implication (residuation) operation is not continuous, then it is *impossible to form a syntactico-semantically complete fuzzy logic with evaluated syntax*. This theorem was originally proved by Pavelka in [63]; see also [53], Theorem 4.2. Consequently, the only plausible structure of truth values on $[0,1]$ for this logic is the standard Łukasiewicz MV-algebra and its isomorphs.

## 3 The Eighties

These years were characterized by increasing interest in the fuzzy set theory and fuzzy logic. A notable paper was that of A. Pultr [69] emphasizing the role of fuzzy equality

[3] In fact, all formulas are then axioms, but most of them only in the degree 0.

in fuzzy set theory where the essential transitivity axiom is stated as follows:

$$(a \sim b) \otimes (b \sim c) \leq a \sim c, \qquad a,b,c \in L.$$

By $x \sim y$, we denote the degree of equality between $x$ and $y$, and $\otimes$ is the multiplication operation in a considered residuated lattice of truth values. This concept has been extensively developed in many papers and books in fuzzy set theory since then. Recall, among other ideas, the important concept of the *real semiset* in AST; i.e. a semisets that is fully characterized by some indiscernibility equivalence relation $\equiv$. The latter is obtained as the intersection of a decreasing countable sequence of set-definable relations[4]

$$\equiv = \bigcap \{R_n \mid n \in \mathbb{FN}\}. \tag{3}$$

This definition is motivated as follows: two objects are indiscernible if all the requirements for still higher and higher precision fail to be met. Hence, fuzzy sets can be seen as reasonable approximations of semisets when one is replacing the indiscernibility relation with the corresponding fuzzy equality. This idea is formally developed in [42].

It seems that the main impulse for the growing interest in fuzzy logic in Czechoslovakia came from the book [43] written by the author of this paper. This book was quite successful and thus was published as a revised English edition in [44] and then again in Czech [46]. It is important to emphasize that the fuzzy set theory is explained in this book in a unified way from the point of view of $\mathrm{Ev}_{Ł}$ (which is described in detail in Chapter 3 of the volume in question) as the metatheory at play. As a result, one can see that the concept of formal fuzzy mathematics as proposed much later (see below) is anticipated in this book.

At the same time, the author of this paper followed the ideas of J. Pavelka and developed a predicate version of $\mathrm{Ev}_{Ł}$ in his PhD thesis defended in 1988. Let us emphasize that the main reviewer of the thesis was again P. Hájek. The 1980s also represented years of intensive discussions between the author of this paper and P. Hájek about fuzzy logic and its meaning.

One part of the thesis was published in two papers [45]. It contains a proof (based on Pavelka's technique) of a generalization of the Gödel-Henkin completeness theorem:

$$T \vdash_a A \quad \text{iff} \quad T \models_a A$$

for all fuzzy theories $T$ and formulas $A$ where by $T \vdash_a A$ we mean that a formula $A$ is *provable in a degree a* in $T$ and by $T \models_a A$, that it is *true in a degree a*. Another crucial property proven there is a generalized deduction theorem:

$$T \cup \{{}^{1}\!/\!_{A}\} \vdash_a B \quad \text{iff} \quad \text{there is } n > 0 \text{ such that } T \vdash_a A^n \Rightarrow B. \tag{4}$$

Note that a slightly weaker form of this theorem is now known as the *local deduction theorem* in fuzzy logic (see below).

[4]Many technical details are omitted here. The interested reader should refer to [73].

It is worth mentioning that a working group for *Fuzzy Sets and Their Applications* was established within the Society of Czechoslovak Mathematicians and Physicists in 1985. This group has had annual meetings since then. The most important of its activities has been the *International Symposium on Fuzzy Approach to Reasoning and Decision-Making* (FARDM'90), which took place in a small town, Bechyně, in June 1990. The main organizers of this event were V. Novák, M. Mareš, J. Ramík, J. Nekola and M. Černý. The symposium had a key influence on the further development of fuzzy logic in Czechoslovakia. On November 17, 1989, the Czechoslovak communistic regime broke down. The country became open, people were allowed to travel freely and thus, closer contact with western scientists could be established. Because the freshly liberated communist country was interesting to visitors, many people took part in FARDM'90, coming both from the other communist countries and from western ones (the number of participants was slightly over 100). For the first time, we had an opportunity to discuss freely and to meet with the people whose papers we rarely had had the opportunity to read before. A large group of Russian scientists came (actually 27); furthermore, there were several people from Hungary, Poland, and East Germany, as well as D. Dubois from France, M. Roubens from Belgium, G. J. Klir, L. Kohout and J. Bezdek from the USA, among others . The impact of this symposium on fuzzy logic in the Czech Republic can still be felt today. Selected papers from this symposium have been published in the edited volume [48].

# 4 The Nineties

## 4.1 Beginning of the decade

This decade was probably the most active and important for the development of fuzzy logic not only in the Czech Republic but also on an international scale. After long and fruitless quarrels between Czech and Slovak representatives that delayed the economical and political progress of Czechoslovakia, it was decided that Czechoslovakia would be split up into two countries: the *Czech Republic* and *Slovakia* (the Slovak Republic). Though many people were regretful, this decision was very useful from the point of view of the development of both countries. At the same time, the existing close relations between scientists in the two nations were not harmed. Concerning fuzzy set theory, while Slovakia was more oriented towards algebraic and analytical aspects of fuzzy set theory, Czech Republic was more oriented towards fuzzy logic and logical foundations of fuzzy mathematics.

The PhD thesis by the author of this paper contained another result: namely a model of natural language semantics using fuzzy logic. The idea of developing such a model was suggested by P. Sgall (head of the Prague linguistic circle) and finally resulted in the book [47][5] that emerged as an unprecedented attempt at applying AST (hence the title) to the theory of linguistic semantics. The reason for choosing AST was the greater philosophical depth and the more sound and better justified tools in

[5]The book was published especially due to the kind support of G. J. Klir.

comparison with fuzzy set theory. While semisets seem to capture the core of the vagueness phenomenon, fuzzy sets in a certain sense seem to quantify it as if from the outside. The book contains a model of the meaning of the most important classes of words and linguistic syntagms. The meaning is represented by special real semisets. Because they are characterized by some indiscernibility relation, they can be technically approximated using fuzzy sets in the sense of [42]. Unfortunately, this book remained almost unnoticed for two reasons in different contexts: it was not mathematical enough and too non-standard for mathematicians, and it was too mathematical for linguists.

This book remained a permanent source of inspiration for the author in using formal fuzzy logic for various kinds of applications, especially when linguistic semantics is at play. The first direction pursued was that of *linguistically oriented control* [49], whose main idea was to develop a technique using which the computer could act as if it understood the expert's linguistic instructions regarding how a given plant should be controlled. The essential difference between this and (classical) fuzzy control is that the expert specifies his description of control directly using words *without thinking about fuzzy sets hidden inside*. This technique has been implemented and applied to control several real processes. The most successful was the control of an aluminium melting furnace in 1996 (see [52])[6].

Another direction was establishing the program of *fuzzy logic in broader sense* in [50]. The main idea was to develop a formal model of natural human reasoning with the emphasis on employing vague semantics of natural language and solving various related practical problems.

### 4.2 Entering of P. Hájek into fuzzy logic

For a long time, P. Hájek had been quite hesitant about fuzzy logic, arguing against it in many discussions with the author of this paper over the course of about 10 years. During one of these discussions in 1990 at the Conference of the Czech Mathematicians and Physicists in Zvíkovské Podhradı, he let himself be persuaded that fuzzy logic is not probability theory in disguise. Another point in favor of fuzzy logic came from G. Takeuti, namely from his paper [71] and from his visit to Prague.

After that time, P. Hájek's name was seen more and more often in connection with fuzzy logic (see, e.g. [18, 19, 30]. In the paper [20], he for the first time clearly formulated his view of fuzzy logic as a special branch of mathematical logic.

The event of crucial importance for the further development of mathematical fuzzy logic was the monograph [27]. In it, P. Hájek established mathematical fuzzy logic as a generalization of classical logic. This logic has slightly modified syntax (through the addition of new connectives and different set of axioms) and non-classical semantics. He actually introduced a class of formal logical systems now called (mathematical)

[6] It should be noted that this application turned out to be very successful and was extended to four more furnaces in subsequent years. The application has already been continuously working for more than 13 years.

*fuzzy logic in narrow sense* (FLn) with traditional syntax. In addition to G. Takeuti, he was inspired mainly by the papers of J. Pavelka, V. Novák, U. Höhle [37, 38], and D. Mundici on MV-algebras (e.g. [41]), as well as S. Gottwald's book on many-valued logic [17], and also by the people developing the mathematical theory of t-norms, namely R. Mesiar, E. P. Klement, E. Pap, D. Butnariu, M. Navara, and others (an essential monograph about t-norms that was published at the turn of the 21st century is [40]).

Fuzzy logic in P. Hájek's terms stems from the assumption that the truth values form a special residuated lattice that then yields rules for the specification of its syntax and semantics. Furthermore, the attempt to coin the interval of reals $[0,1]$ as a set of truth values led him to establish his *basic fuzzy logic* (BL-fuzzy logic, or simply BL). Based on the Moster-Shields theorem for t-norms stating that each continuous t-norm is either isomorphic to minimum, product, or Łukasiewicz conjunction (which is also a t-norm) or otherwise an ordinal sum of them (see [40] for details), he could confine to four kinds of main fuzzy logic: *Gödel logic*, which was initiated by K. Gödel and further developed by G. Takeuti in [71]; *Łukasiewicz logic*, which has been studied already for many years; *product logic*, which was established as a new logic by P. Hájek; and BL-fuzzy logic, comprised of all three previous ones. He also recognized the essential role of the prelinearity axiom

$$(a \to b) \vee (b \to a) = \mathbf{1}$$

which then enabled him to prove the following *completeness theorem* for all of the above-discussed logics: For every formula, $A$ the following is equivalent:

1. $A$ is provable.
2. $A$ is a $\mathbf{1}$-tautology w.r.t. the corresponding algebras of truth values.
3. $A$ is a $\mathbf{1}$-tautology w.r.t. the corresponding *linearly ordered* algebras of truth values.

This theorem supports the idea that the truth values of fuzzy logics should always be linearly ordered. The famous *standard completeness theorem* stating that semantics of BL-fuzzy logic can be confined only to BL-algebras on $[0,1]$ in item (iii) (cf. [8]) is a successful culmination of P. Hájek's ideas.

As for $\mathrm{Ev}_{Ł}$, P. Hájek never accepted the idea of evaluated syntax; thus, this logic was not for him a "genuine" logic. He reformulated it inside Łukasiewicz logic as a side logic among other fuzzy ones and calls it *Rational Pavelka Logic*. In a more or- ess artificial way, it is possible to introduce a special degree called the *provability degree*. The author of this paper, however, does not think that this term is convenient because it occludes the essential idea that axioms need not be fully convincing and thus that evaluated syntax is obtained as a nice generalization of the classical one. Namely, it can be demonstrated (cf. [53]) that the concept of provability degree can be seen as a natural generalization of classical provability and, consequently, that traditional

syntax is a special case of the evaluated one. From this point of view, J. Pavelka left the realm of classical logic in a most radical way, thus introducing a new kind of logic. On the other hand, there is only one $\mathrm{Ev}_{Ł}$ (cf. our discussion on $\mathrm{Ev}_{Ł}$ above), thus it seems to be too narrow for many mathematicians.

### 4.3 Ending of the decade

P. Hájek's monograph is the first of a series of books providing, in our opinion, a full picture of the state of the art in fuzzy logic in the nineties. The second book is [53], written by V. Novák, I. Perfilieva and J. Močkoř. It contains a complete presentation of $\mathrm{Ev}_{Ł}$. In addition, the book also contains detailed elaboration of the approximation properties of fuzzy models, the model theory of fuzzy logics in categories, and a more detailed elaboration of FLb (at that time, developed on the basis of $\mathrm{Ev}_{Ł}$ only). A notable result by I. Perfilieva is an original constructive proof of the McNaughton theorem for Łukasiewicz logic, which states that each formula of the latter can be uniquely represented by a piecewise linear function on $[0, 1]$.

The third book of this series is the volume [54] edited by V. Novák and I. Perfilieva. It contains contributions by several authors[7] and is focused on various further aspects of fuzzy logic not covered by the previous two books. A full picture of the state of the art of mathematical fuzzy logic in $20^{\text{th}}$ century is completed by the book [40] on t-norms written by E. P. Klement, R. Mesiar and E. Pap.

The successful development of fuzzy logic in the Czech Republic also opened the door to an institutional establishment. Namely, in 1996, the Institute for Research and Applications of Fuzzy Modeling was established in the University of Ostrava in Ostrava (IRAFM OU). Its founders were V. Novák, J. Močkoř, A. Dvořák, R. Bělohlávek, R. Smolíková and D. Jedelský. It became a renowned place of work that successfully combines theoretical and applied research in one place.

In 1999, IRAFM OU was significantly strengthened by Russian mathematician I. Perfilieva, who naturalized in the Czech Republic. She introduced many new impulses and, in addition to her work in fuzzy logic, she introduced an interesting and related area: the *fuzzy approximation* of functions.

At the turn of the century, a group of young people gathered around P. Hájek, mostly at the Institute of Computer Science of the Academy of Sciences of the Czech Republic in Prague (ICS AV ČR). Among them, very active people include P. Cintula and L. Běhounek, R. Horčík, Z. Haniková, D. Coufal and T. Kroupa (he works at the Inst. of Information Theory and Automation of the ASCR).

Both places of work became leading centers of research in fuzzy logic, not only in the Czech Republic, but also internationally Their researchers meet regularly several times per year and organize various meetings and conferences. The most important

[7] The contributiors were: L. A. Zadeh, G. J. Klir, E. P. Klement, R. Mesiar, E. Pap, J. Paris, A. Di Nola, G. Georgescu, I. Leuştean, E. Trillas, A. R. de Soto, S. Cubillo, O. Hájek, G. Gerla, S. Lehmke, E. Turunen, V. Novák, I. Perfilieva, H. Thiele, D. Mundici, F. Esteva, P. Garcia, L. Godo, S. Gottwald, F. Klawonn, R. Bělohlávek, K. Broda, A. Russo, D. Gabbay

event in the nineties was the Seventh IFSA'97 (International Fuzzy Systems Association) World Congress, which took place in June 25-29, 1997 and was co-organized by the University of Ostrava, the School of Economics in Prague, the Institute of Computer Science ASCR, the Institute of Theory of Information and Automation ASCR in Prague, Czech the Technical University in Prague, Slovak Technical University in Bratislava, and Silesian University in Karviná. The general chair was V. Novák, and the other organizers were J. Ivánek, M. Mareš, R. Mesiar, P. Hájek, J. Ramík, P. Vysoký, P. Přibyl, and M. Zeithamlová. The IFSA'97 World Congress in Prague was evaluated by many of its participants as the best of all IFSA congresses. The proceedings was published by the famous publishing house Academia in Prague [51].

# 5 21st Century

## 5.1 P. Hájek and his group in fuzzy logic

Thanks to P. Hájek's monograph (and also to the other mentioned books), the beginning of 21st was devoted to the intensive study of mathematical fuzzy logic not only in the Czech Republic but also in several other European countries, especially Italy and Spain. The results penetrated to a great depth both in the corresponding algebraic structures of truth values and in the properties of the corresponding formal logical systems. Various other logics have been studied, for example, ΠMTL by R. Horčík in [35, 36]. The work led to the combination of various axioms and was concluded in the so-called ŁΠ fuzzy logic initiated by Spanish and Italian colleagues in [16]. Their logic has been extended to a predicate first-order version and elaborated on in detail by P. Cintula in [9, 10].

One of the problems intensively studied by P. Hájek since the 1990s is complexity of fuzzy logics. This study was begun in [21] and then continued in a sequence of papers [22–26]. The most recent one is [12]. The complexity properties of fuzzy logic are very intricate and depend on the chosen class of logics and their semantics. The following sets of formulas have been studied: 1-tautologies, 1-satisfiable formulas (i.e., those that are 1-true for at least one model), positive tautologies (true in a degree $> 0$) and positively satisfiable formulas. Their complexity ranges, in general, from $\Sigma_1$ to $\Pi_2$, but in some cases, it is even non-arithmetical (usually if the given logic contains the product connective: e.g., BL). For example, Łukasiewicz predicate logic with respect to the standard algebra (2) has $\Pi_2$-complete set of 1-tautologies, $\Pi_1$-complete set of 1-satisfiable formulas, $\Sigma_1$-complete set of positive tautologies and $\Sigma_1$ complete set of positively satisfiable formulas. If we consider general tautologies (i.e., tautologies with respect to linearly ordered algebras from the given variety) then the given fuzzy logic is only $\Sigma_1$-complete. Moreover, for each set $\mathcal{K}$ of continuous t-norms such that propositional $\mathcal{K}$-tautologies are axiomatizable, general tautologies with respect to $\mathcal{K}$ are $\Sigma_1$-complete (cf. [25]). It is not surprising that complexity of fuzzy logics is sometimes fairly high given that fuzzy logics should be used in modeling various complex phenomena, namely if vagueness is at play and one cannot expect miracles. Thus, to apply it in practice, we must seek special local solutions.

Various kinds of fuzzy logics and their structure have been extensively studied. The most comprehensive study is the paper [13] written by P. Hájek, P. Cintula and R. Horčík, who studied 60 various systems of fuzzy logic simultaneously. A similar comprehensive paper on algebraic semantics of fuzzy logic is [14].

The fact that there are so many systems of fuzzy logic has inspired discussion about how fuzzy logic can be defined. In [11], P. Cintula proposes to delineate fuzzy logic mathematically, namely as a logic of the comparative notion of truth. A special issue entitled "What is Fuzzy Logic" from the journal Fuzzy Sets and Systems (Volume 157, Issue 5, Pages 595-718, March 2006) also contains the opinion of P. Hájek's group in the papers [5,28]. This discussion led them to the concept of *core fuzzy logic* (see [29]): it is a logic that is an expansion of MTL logic.fulfiling

$$A \Leftrightarrow B \vdash C(A) \Leftrightarrow C(B),$$

(for all formulas $A, B, C$) and holding the *local deduction theorem* (cf. (4)):

$$T \cup \{A\} \vdash B \quad \text{iff} \quad \text{there is } n > 0 \text{ such that } T \vdash A^n \Rightarrow B$$

(for all formulas $A, B$). One of the problems coming out of a formal definition is the following: one can argue that not all logics fulfilling it are indeed capable of providing a model of the vagueness phenomenon (e.g., the Gödel logic). This leads to an informal, taxative definition —fuzzy logic is one of several explicitly named logics chosen from the point of view of their capability to provide a model of essential features of the vagueness phenomenon and the human way of reasoning. A contribution to this direction is presented in [55,59].

A special direction that is very important for the development of fuzzy set theory is the program of *formal fuzzy mathematics* introduced by P. Cintula and L. Běhounek in [4]. Their idea is to develop various areas of fuzzy mathematics formally using mathematical fuzzy logic in the style introduced by P. Hájek as the metamathematical frame. The mathematical background, which is based on the class of fuzzy logics developed before, is the *fuzzy class theory* [3]. In a sense, this is a somewhat restricted higher-order fuzzy logic that might be equivalent with the fuzzy type theory mentioned below. Recall that the program of formal fuzzy mathematics is a precise formulation of the program anticipated already in the book [44].

P. Cintula and L. Běhounek not only introduced the program of fuzzy mathematics but also established a group of people (T. Kroupa, R. Horčík, M. Daňková, C. Noguera and few other ones) who began to realize it step by step. In any case, this is a significant attempt to put fuzzy set theory on very well and strongly established grounds.

## 5.2 Other activities in fuzzy logic in the Czech Republic

Several PhD theses have been defended dealing with various aspects of fuzzy logic, considerig the fundamental issues (P. Cintula), algebraic aspects (R. Horčík, Z. Haniková),

generalized quantifiers (M. Holčapek), normal forms of approximation theory (M. Daňková) and applications (A. Dvořák, M. Štěpnička).

The work of IRAFM OU in fuzzy logic has focused on several aspects. In 2005, higher-order fuzzy logic was introduced by V. Novák in [56]. In accordance with the classical type theory[8] it is called the *fuzzy type theory*. In principle, the λ-calculus has been extended and many-valued semantics has been introduced for it. The main motivation was to introduce a sound formal means for the development of FLb, which would be more convenient than $\mathrm{Ev}_{Ł}$ as applied so far. Let us remark that everything formulated in formal fuzzy mathematics using the fuzzy class theory can also easily be formulated in the fuzzy type theory (using the λ-calculus).

Among the topics which were elaborated more deeply, let us mention the concept of *fuzzy logic normal forms* in predicate fuzzy logic and their use in the fuzzy approximation: i.e., a formal theory of approximation of functions on the basis of imprecise information. This concept was initiated by I. Perfilieva already in [65] and elaborated on further by M. Daňková (cf. [15, 64, 66, 67]).

In 2006, the Russian edition of the book by V. Novák, I. Perfilieva and J. Močkoř was published in [58]. The book has been quite successful in this vast market. Notable results have been obtained also in the theory of generalized (fuzzy) quantifiers by M. Holčapek and A. Dvořák [33, 34], who introduced the results of the "classical theory" of generalized quantifiers (cf. [68]) into mathematical fuzzy logic, and V. Novák [57], who proposed a formal model of so-called intermediate quantifiers (e.g., "most, many, a lot of, several, a few", etc.).

Let us also mention the attempt at leaving the realm of residuated lattice-based fuzzy logic by introducing a new special algebraic structure called EQ-algebra, in which the fundamental operation is that of *fuzzy equality* $\sim$. The implication in it is a derived operation (cf. [60–62]) defined by

$$a \to b = (a \wedge b) \sim a. \tag{5}$$

This direction in some sense follows the ideas of G. W. Lebniz, who claimed that "a fully satisfactory logical calculus must be an equational one". As a matter of fact, formula (5) also follows the Leibnitz idea[9].

Another small group led by R. Bělohlávek (he left IRAFM OU in 2004) was established in Olomouc. Let us refer especially to two books [2] and [6] where the fuzzy equational logic has been established. It has been shown in this book that when confining only to equalities, generalized completeness in the sense of $\mathrm{Ev}_{Ł}$ can be obtained even without the assumption that the implication operation is continuous on $[0,1]$.

Several scientific meetings have taken place in the Czech Republic: in 2005, the conference "The Logic of Soft Computing V" was organized in Ostrava by IRAFM

[8] The type theory was introduced already by B. Russel and A. Whitehead in their Principia Mathematica; it was later on developed more formally by A. Church in [7] and elaborated in detail by L. Henkin in [31,32] and P. Andrews in [1].

[9] Let $A \equiv B$ be an equality of concepts and $AB$ their intersection. Then Leibnitz interprets that "if $A$ then $B$" as $AB \equiv A$.

OU. The conference was the penultimate of the series of this kind of conferences on formal fuzzy logic[10]. A special issue of some selected papers has been published in Fuzzy Sets and Systems vol. 158, 2007.

An attempt at organizing a meeting of philosophers and mathematicians in one place was "Conference on Uncertainty & Vagueness" which took place in Prague, September 5-8, 2006. The main goal of this conference was to make philosophers familiar with mathematical fuzzy logic and to explain to them that their criticism of the latter is unjustified (cf. [39]). Unfortunately, it seems that the philosophers (though hopefully not all of them) are to a great extent closed towards ideas coming from outside and thus, from this point of view, the conference was not fully successful.

The most important meeting in the first decade of the 21st century was the $5^{th}$ EUSFLAT Conference (European Society for Fuzzy Logic and Technology), which took place in Ostrava (Czech Republic) during September 11-14, 2007. The organizer of this conference was IRAFM OU. The conference was successful and the proceedings were published in [70]. In this conference, the Working Group on Mathematical Fuzzy Logic (MathFuzzLog) of EUSFLAT was established. It aims to conduct and promote research in Mathematical Fuzzy Logic.

It is difficult to write a complete presentation of the results related fuzzy logic in the Czech Republic in 21st century because many activities of various kinds have been accomplished and so, much more space than we have would be needed. We refer the readers to the webpages http:\\irafm.osu.cz\ and http:\\www.cs.cas.cz\mathfuzzlog\ where more information, including full references to many publications or to their preprints, can be found.

## 6 Conclusion

This paper aims especially to pay tribute to P. Hájek and his work in fuzzy logic on the occasion of 70th birthday celebration. We have briefly overviewed some of the main points in the development of fuzzy logic in Czechoslovakia and, since 1993, in the Czech Republic. Our presentation is far from being complete because we focused mainly on fuzzy logic and because we omitted a lot of results obtained in other areas of fuzzy mathematics such as fuzzy mathematical programming and other ones.

Our goal was to show that the contribution of P. Hájek to fuzzy logic is crucial. Without him, the subject would be incomparably less developed. He deserves many thanks for his work. We wish him good health and many inspiring ideas in the following years.

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[10]The first one was organized in Capri in 2001 and the last one in Malaga in 2006.

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# A Concise Glance at the History of Fuzzy Logic in Czechia (with a glimpse of the origin of data-mining — the GUHA method)

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**Abstract** In this paper, we will briefly look at the history of mathematical fuzzy logic in Czechoslovakia (and the Czech Republic) starting from seventies of the twentieth century until 2009. We will touch also the alternative set theory as well as history of the GUHA method.

## 1 Introduction

This paper is a brief look at the history of mathematical fuzzy logic in Czechoslovakia starting from seventies of the twentieth century until today (June 2009). It must be emphasized that in 1993, Czechoslovakia was splitted up and the Czech Republic and Slovak Republic were established instead. Therefore, starting from that year, we will refer mostly to mathematical fuzzy logic in the Czech Republic.

The text is based on personal remembrances and the author claims neither completeness nor objectivity. The list of publications presented according to years of appearance only accompanies our description and it is far from being complete.

We focus mostly to mathematical fuzzy logic and some closely related topics. Therefore, various other branches intensively studied in fuzzy set theory as well as in logic are not included. Besides fuzzy logic, we will also mention the alternative set theory and the GUHA method.

It should be noted that some areas which we suppose to be less known are discussed in more detail. Finally, let us remark that time to time we accompany our memories by mathematical concepts or formulas to provide a more clear or specific explanation but, because of different focus of this paper, we cannot be too detailed and exact.

## 2 The Seventies

There was not a significant movement in the life in Czechoslovakia in the seventies. These years have been negatively influenced by the communist normalization after the year 1968 and so, those who stayed in Czechoslovakia[1] were subject to various kinds of restrictions motivated by endeavor of the communist party to have a voice in everything. Especially serious was this pressure in the humanities. Luckily, in mathematics this was not so strong because orthodox enthusiastic communists were not scientists and so, they could not understand abstract sciences such as mathematics. On the other hand, the admission to western literature was quite limited, mainly for the economical reasons rather than political ones; Czechoslovakia simply had very limited sources of western currency and so, the research was partly closed towards western science. Luckily, a lot of good western books were available in Russian because they were translated from English in Soviet Union.

Surprisingly, two great ideas (at least) were raised in logic and set theory in these gray days. The first idea was the concept of the, so called, *Alternative Set Theory* (AST), and the second idea was establishing a precise formal system of fuzzy logic $\mathrm{Ev}_{\mathrm{Ł}}$ which is now known as *fuzzy logic with evaluated syntax*.

The main person in AST was P. Vopěnka who, at about 1974, gathered a group of logicians[2] who commenced developing a new set theory that might be alternative (hence the name) to the classical Cantorian one (ST).

[1] Many thousands of mostly well educated people emigrated from Czechoslovakia in 1968–69.

[2] The other important people in this group were K. Čuda, A. Sochor and J. Mlček.

There are several important principles of AST which depart from ST. The most important is a new model of infinity. While the classical set theory constructs infinity by never-ending extension of a set, in AST the infinity emerges when dealing with sufficiently large but otherwise classically finite set. We arrive at the concept of the, so called, *natural infinity* which, unlike ST, is directed inside the set. This is possible by introducing a special mathematical object which is related also to fuzzy logic, namely that of a *semiset*. A semiset is a proper class (not a set) $X$ being a subclass of some set $a$, i.e.

$$X \subseteq a.$$

Note that semisets are by no means as strange as it might seem from the first sight. For example, let us consider a class *Sm* of *small natural numbers*. Such numbers are 0, 1, ... but this sequence is not finished — there is no last small number and, therefore, also no first large number. On the other hand there is surely a number $n$ which large (it is not small), for example $n = 10^{10}$. Thus, we have $Sm \subseteq n$. Let us stress that *Sm* cannot be a set because then it would have to be finite (since $n$ is finite) and so, there would have to exist the last small number. The class *Sm* is a typical semiset and it is a subclass of a semiset $\mathbb{FN}$ of all *finite natural numbers* which is in AST defined as a class of all well ordered sets that do not contain a proper subclass. The semiset $\mathbb{FN}$ is a proper subclass of a *class* $\mathbb{N}$ *of all natural numbers*. This means that any number $\alpha \in \mathbb{N} - \mathbb{FN}$ is infinite. But recall again that such a number $\alpha$ is *classically finite* (though very large in the given context). We encountered infinity phenomenon emerging inside very large sets. The semiset $\mathbb{FN}$ is *infinitely countable* while $\mathbb{N}$ is *uncountable*.

Let us remark that semisets can be consistently introduced even inside classical Gödel-Bernays axiomatic set theory by weakening slightly its axioms. This has been done in a strictly formal way in the book by P. Vopěnka and P. Hájek [5].

Except for many papers published mostly in the journal CMUC (Commentationes Mathematicae Universitatis Carolinae), the book [9] was published presenting the essential results of AST. This book was small and brief. A much more extensive book presenting a lot of well established philosophical backgrounds is the book [16]. Unfortunately, it has never been translated into English. The AST theory has been developed until the beginning of nineties. The most recent book on AST focusing especially on its philosophy is [72].

The concept of semiset is closely related to the main topic of this paper — the fuzzy logic. Indeed, the reader has surely noticed that elements of the above mentioned semiset *Sm* represent numbers of grains which do not form a heap in the known *sorites paradox*:

> *One grain does not form a heap. Adding one grain to what is not yet a heap does not make a heap. Consequently, there are no heaps.*

This paradox is refuted in AST. Moreover, semisets represent vaguely defined classes, i.e. just the objects which are in fuzzy logic modeled by *fuzzy sets*.

The fuzzy logic with evaluated syntax $\mathrm{Ev}_{Ł}$ has been developed by J. Pavelka in his PhD thesis defended in 1976 and published in a sequence of three papers [8]. These papers contain full description of the formal system of propositional $\mathrm{Ev}_{Ł}$ including algebraic analysis of possible structures of its truth values and its metatheory. The third of the the papers [8] contains a complicated algebraic proof of syntactico-semantical completeness of $\mathrm{Ev}_{Ł}$.

The second of the papers [8] contains a deep algebraic analysis of the possible structures of truth values and refers especially to the concept of residuated lattice, thus anticipating the essential consequences in the theory of fuzzy logic developed later by P. Hájek. The basic considered algebra there is (what is now called as) *standard Łukasiewicz MV-algebra*

$$\langle [0,1], \vee, \wedge, \otimes, \rightarrow, 0, 1 \rangle$$

where $\otimes$ is the Łukasiewicz conjunction (also called Łukasiewicz or bold product) defined by $a \otimes b = \max(0, a+b-1)$ and $\rightarrow$ is the Łukasiewicz implication defined by $a \rightarrow b = \min(1, 1-a+b)$, $a, b \in [0,1]$.

This logic goes thoroughly along the principal concept of fuzziness by assuming not only truth values between 0 and 1 but enabling also axioms not to be fully convincing, i.e. a formula $A$ can be axiom only in some (arbitrary) degree. From it follows the concept of proof having a value which expresses the degree in which the proof is convincing. As a consequence, we arrive at the concept of provability degree of a formula $A$. The prevailing principle in $\mathrm{Ev}_{Ł}$ is the *maximality principle* stating that if a formula $A$ is assigned more truth values then the final truth value is equal to the maximum (supremum) of all of the former ones.

A very important is the metatheorem stating that if we consider a residuated lattice on $[0,1]$ and the implication (residuation) operation is not continuous then it is *not possible to form a syntactico-semantically complete fuzzy logic with evaluated syntax*. Consequently, the only plausible structure of truth values on $[0,1]$ for this logic are the standard Łukasiewicz MV-algebra and its isomorphs.

## 3 The Eighties

In the eighties, we can encounter an increasing interest in fuzzy set theory and, consequently, in fuzzy logic. A notable paper is that of A. Pultr [11] raising up the role of fuzzy equality in fuzzy set theory where the essential transitivity axiom is stated as follows:

$$(a \sim b) \otimes (b \sim c) \leq a \sim c, \qquad a, b, c \in L$$

where $x \sim y$ is the degree of equality and $\otimes$ is the multiplication operation in a considered residuated lattice of truth values. This concept has been extensively elaborated in many papers and books in fuzzy set theory since then. Among others, recall that one of the motivations of the concept of semiset is close to the motivation of the concept of fuzzy set. Namely, important role in AST is played by the, so called, *real semisets* which are fully characterized by some indiscernibility equivalence relation $\equiv$. The

latter is obtained as intersection of a decreasing countable sequence of set-definable relations[3]

$$\equiv = \bigcap \{R_n \mid n \in \mathbb{FN}\}. \tag{1}$$

This definition is motivated as follows: two objects are indiscernible if all the still more and more precise criteria fail. Thence, fuzzy sets can be seen as reasonable approximations of semisets when replacing the indiscernibility relation by the corresponding fuzzy equality. This idea is formally elaborated in [10].

It seems that the main impulse for the growing interest in fuzzy logic in Czechoslovakia came from the book written by the author of this paper [12]. This book was quite successful and so, it was published as revised English edition in [15], and then again in Czech [18]. It is important to emphasize that the fuzzy set theory has been explained in this book in a unified way from the point of view of $\mathrm{Ev}_{Ł}$ (which is in detail described in its Chapter 3) as its metatheory and so, the concept of formal fuzzy mathematics proposed much later (see below) has been anticipated in this book.

At the same time, the author of this paper followed the ideas of J. Pavelka and developed a predicate version of $\mathrm{Ev}_{Ł}$ in his PhD thesis defended in 1988. This work contains proof (based on Pavelka's technique) of a generalization of the Gödel-Henkin completeness theorem:

$$T \vdash_a A \quad \text{iff} \quad T \models_a A$$

for all fuzzy theories $T$ and formulas $A$ where by $T \vdash_a A$ we mean that a formula $A$ is *provable in a degree a* in $T$ and by $T \models_a A$, that it is *true in a degree a*. Another crucial property proved there is a generalized deduction theorem:

$$T \cup \{{}^{1}\!/\!_{A}\} \vdash_a B \quad \text{iff} \quad \text{there is } n > 0 \text{ such that } T \vdash_a A^n \Rightarrow B.$$

Note that a slightly weaker form of this theorem is now known as *local deduction theorem* in fuzzy logic. The predicate $\mathrm{Ev}_{Ł}$ has been published in two papers [17] and also included in the revised versions of the above mentioned book [15, 18].

It is worth mentioning that a working group for *Fuzzy Sets and Their Applications* was established within Society of Czechoslovak Mathematicians and Physicists in 1985. This group had annual meetings since and its activities have been crowned by organizing the *International Symposium on Fuzzy Approach to Reasoning and Decision-Making* (FARDM'90) which took place in a small town Bechyně in June 1990. The main organizers of this event were V. Novák, M. Mareš, J. Ramík, J. Nekola and M. Černý. The symposium had essential influence on further development of fuzzy logic in Czechoslovakia.

First of all, remember that (November) 1989 was the year when the communistic regime has broken down. Therefore, the country became open, people were allowed to travel freely and thus, closer contacts with western scientists could be established. Since freshly freed communist country was interesting for visiting, many people took part at FARDM'90 coming both from the other communists countries as well as from

[3] Many technical details are omitted. The interested reader is referred to [9].

western ones (the number of participants was slightly over 100). For the first time, we had an opportunity to discuss freely and to meet with people whose papers we scarcely could read before. Let us remember that there was a large group of Russian scientists (actually 27), several people from Hungary, Poland, East Germany but also D. Dubois from France, M. Roubens from Belgium, G. J. Klir, L. Kohout and J. Bezdek from USA and few other people. The impact of this symposium on the Czech fuzzy logic can still be felt even up to these days. Selected papers from this symposium have been published in the edited volume [21].

## 4 The Nineties

This decade was probably the most active and important for the development of fuzzy logic not only in the Czech Republic but on the international scale. After long and barren quarrels between Czech an Slovak representatives that stuck the economical and political progress of Czechoslovakia, it was decided that Czechoslovakia would be split up into two countries: *Czech Republic* and *Slovakia* (Slovak Republic). Though many people were regretful, this decision was very useful from the point of view of the development of both countries. The existing close relations between scientists of both nations were not broken. Concerning fuzzy set theory, it could be seen more clearly that while Slovakia has been more oriented towards algebraic and analytical aspects of fuzzy set theory, Czech Republic has been more oriented towards fuzzy logic and logical foundations of fuzzy mathematics.

The PhD thesis of the author of this paper contained another result, namely a model of natural language semantics using fuzzy logic. The hint to develop such a model was suggested by P. Sgall (head of the Prague linguistic circle) and finally resulted in the book [20][4] that became an unprecedented attempt at applying AST (hence the title) to the theory of linguistic semantics. The reason for this decision was greater philosophical depth and more sound and better justified tools of AST in comparison with the fuzzy set theory. While semisets seem to capture the core of the vagueness phenomenon, fuzzy sets in a certain sense seem to quantify it as if from the outside. Thus, the meaning of the most important classes of words and linguistic syntagms is modeled using special real semisets. Since these are characterized by some indiscernibility relation, they can be technically approximated using fuzzy sets in the sense of [10]. Unfortunately, this book remained almost unnoticed for two reasons: it was too little mathematical and non-standard for mathematicians, and too mathematical for linguists.

This book remained a permanent source of inspiration for the author to involve formal fuzzy logic into various kinds of applications, especially when linguistic semantics is at play. The first direction was *linguistically oriented control* [28] whose main idea was to develop a technique on the basis of which the computer could behave as if understanding the expert's linguistic instructions how the given plant should be

[4]The book was published especially due to the kind support of G. J. Klir.

controlled. The essential difference from the (classical) fuzzy control is that the expert specifies his description of control directly using words *without thinking about fuzzy sets hidden inside*. This technique has been implemented and applied to control several real processes. The most successful has been control of one aluminium melting furnace in 1996 (see [38])[5].

Another direction was establishing the program of *fuzzy logic in broader sense* in [29]. Its main idea was to develop a formal model of natural human reasoning with the stress on employing vague semantics of natural language and to solve various related practical problems.

The event of crucial impact on further development of mathematical fuzzy logic was P. Hájek's monography [32]. It should be noted that before, P. Hájek had been quite reserved towards fuzzy logic and argued against it in many discussions held with the author of this paper for about 10 years. After all, he changed his opinion in about 1990 when he realized that fuzzy logic is not the probability theory in disguise (for many arguments in this respect, see [57]). The second hint in favor of fuzzy logic was visit of G. Takeuti in Prague and his paper [22].

In his monograph, P. Hájek established mathematical fuzzy logic as a generalization of classical logic with modified syntax and non-classical semantics. We arrive at the class of formal logical systems now called *fuzzy logic in narrow sense* (FLn). Except for G. Takeuti, he was inspired mainly by the papers of J. Pavelka, V. Novák, U. Höhle [26, 27], D. Mundici on MV-algebras (e.g. [24]), S. Gottwald's book on many-valued logic [13], and also by the people developing mathematical theory of t-norms, namely R. Mesiar, E. P. Klement, E. Pap, D. Butnariu, M. Navara, and other ones (essential monograph about t-norms which was published at the turn of the century is [37]).

Fuzzy logic in P. Hájek's conception stems from the assumption that truth values form a special residuated lattice which then gives rules for specification of its syntax and semantics. Furthermore, the attempt to coin the interval of reals $[0,1]$ as a set of truth values led him to establishing his *basic fuzzy logic* (BL-fuzzy logic or simply BL). Based on the Moster-Shields theorem for t-norms stating that each continuous t-norm is isomorphic either to minimum, or to product, or to Łukasiewicz conjunction (which is also a t-norm), or it is an ordinal sum of them (see [37] for the details) he could confine to four kinds of main fuzzy logics, namely *Gödel logic*, which was initiated by K. Gödel and further developed by G. Takeuti in [22], *Łukasiewicz logic*, which has been studied already for a long time before, *product logic*, which was established as a new logic by P. Hájek, and BL-fuzzy logic comprising all the three previous ones. He also recognized essential role of the prelinearity axiom

$$(a \to b) \vee (b \to a) = \mathbf{1}$$

which then enabled to prove the following *completeness theorem* for all the above

[5] It should be noted that this application turned out to be very successful and it was extended to five other furnaces in the subsequent years. The application has already been continuously working for more than 13 years.

discussed logics: For every formula $A$ the following is equivalent:

1. $A$ is provable.
2. $A$ is a $\mathbf{1}$-tautology w.r.t. the corresponding algebras of truth values.
3. $A$ is a $\mathbf{1}$-tautology w.r.t. the corresponding *linearly ordered* algebras of truth values.

This theorem supported the idea that the truth values of fuzzy logics should always be linearly ordered. The famous *standard completeness theorem* stating that in BL-fuzzy logic we can confine only to BL-algebras on $[0, 1]$ in item (iii) (cf. [35]) is a successful crown of P. Hájek's ideas.

As for $\mathrm{Ev}_{Ł}$, P. Hájek never accepted the idea of evaluated syntax and so, this logic was not for him a "genuine" logic. He reformulated it inside Łukasiewicz logic as a side logic among other fuzzy ones and calls it *Rational Pavelka Logic*. In a more-or-less artificial way, it is possible to introduce a special degree called *provability degree*. The author of this paper, however, does not think that this term is convenient because it hides the essential idea that axioms need not be fully convincing and thus, that evaluated syntax is obtained as a nice generalization of the classical one. Namely, it can be demonstrated (cf. [33]) that the concept of provability degree can be seen as a natural generalization of the classical provability and, consequently, that traditional syntax is a special case of the evaluated one. From this point of view, J. Pavelka left the realm of classical logic in a most radical way thus introducing a new kind of logic. On the other hand there is only one $\mathrm{Ev}_{Ł}$ (cf. our discussion on $\mathrm{Ev}_{Ł}$ above) and so, it seems to be too narrow for many mathematicians.

A complete presentation of $\mathrm{Ev}_{Ł}$ is contained in the book [33] written by V. Novák, I. Perfilieva and J. Močkoř. Besides $\mathrm{Ev}_{Ł}$, the book contains also a detailed elaboration of approximation properties of fuzzy models, model theory of fuzzy logics in categories, and a more detailed elaboration of FLb (in that time, developed on the basis of $\mathrm{Ev}_{Ł}$ only).

The books [32, 33] together with the edited volume [39], and the book [37] on t-norms provide a full picture of the state of the art in fuzzy logic in nineties.

The successful development of fuzzy logic in the Czech Republic opened the door also to an institutional establishment. Namely, in 1996, the Institute for Research and Applications of Fuzzy Modeling has been established in the University of Ostrava in Ostrava (IRAFM OU). Its founders were V. Novák, J. Močkoř, A. Dvořák, R. Bělohlávek, R. Smolíková and D. Jedelský. Since then, it became a renowned place of work which successfully combines theoretical and applied research on one place.

In 1999, IRAFM OU has been significantly strengthened by Russian mathematician I. Perfilieva who naturalized in the Czech Republic. She brought many new impulses and besides fuzzy logic, she introduced an interesting and related area of *fuzzy approximation* of functions. As a coauthor of the book [33], she gave an original constructive proof of McNaughton theorem for Łukasiewicz logic which states that

each formula of the latter can be uniquely represented by a piecewise linear function on $[0,1]$.

At the turn of the century, a group of young people gathered around P. Hájek in the Institute of Computer Science of the Academy of Sciences of the Czech Republic in Prague (ICS AV ČR). Among them, the most active are P. Cintula and L. Běhounek. Among the other ones, let us name R. Horčík, Z. Haniková and D. Coufal.

Both places of work became leaders of the research in fuzzy logic not only in the Czech Republic for the 21st century but in international measures. Their workers meet regularly and organize together various meetings and conferences.

The most important event in nineties was organization of the Seventh IFSA (International Fuzzy Systems Association) World Congress which took place in June 25–29, 1997 and was co-organized by the University of Ostrava, School of Economics in Prague, ICS AV ČR, Institute of Theory of Information and Automation AV ČR in Prague, Czech Technical University in Prague, Slovak Technical University in Bratislava, and Silesian University in Karviná. The general chair was V. Novák, the other organizers were J. Ivánek, M. Mareš, R. Mesiar, P. Hájek, J. Ramík, P. Vysoký, P. Přibyl, and M. Zeithamlová. The IFSA'97 World Congress in Prague was evaluated by many of its participants as the best of all IFSA Congresses. The proceedings was published by the famous publishing house Academia in Prague [31].

## 5 21st Century

The previous decade opened the door to intensive study of mathematical fuzzy logic which penetrated into a great depth both in the corresponding algebraic structures of truth values as well as properties of the formal logical systems. Various other logics, except for the four ones introduced in P. Hájek's book were studied, for example [51, 55]. It seems that practically every residuated algebra provides a corresponding logic which is syntactico-semantically complete. The work went to combination of various axioms and was concluded in the, so called ŁΠ fuzzy logics initiated by Spanish and Italian colleagues in [41]. Their logic has been extended to predicate version and elaborated in detail by P. Cintula in [40, 49].

Let us also mention the attempt at leaving the realm of fuzzy logics based on residuated lattices by introducing a special algebraic structure called EQ-algebra where the fundamental operation is that of *fuzzy equality* while implication is derived (cf. [69–71]). This direction in some sense follows ideas of G. W. Lebniz who claimed that "a fully satisfactory logical calculus must be an equational one".

One of the problems studied by P. Hájek since nineties are complexity properties of fuzzy logics. This study has started in [25] and then continued in several papers [30, 42]. The recent one is [68]. It is not surprising that the complexity of most of fuzzy logics is very high, even non-arithmetical (e.g. the predicate product logic).

Several PhD theses have been defended dealing with various aspects of fuzzy logics from the fundamental issues (P. Cintula), algebraic aspects (R. Horčík, Z. Haniková),

generalized quantifiers (M. Holčapek), normal forms for approximation theory (M. Daňková), up to its applications (A. Dvořák, M. Štěpnička).

The study of various kinds of fuzzy logics continued. For example, in [65], 60 various logical systems have been studied at the same time. This fact raised questions about which logical system is the real fuzzy logic. Partial answers from different point of view have been given by P. Hájek in [62], V. Novák in [64], or L. Běhounek and P. Cintula in [61]. The full agreement, however, has not been achieved.

The work in IRAFM OU has been focused, besides others, to the concept of fuzzy logic normal forms and their use in fuzzy approximation — the concept initiated by I. Perfilieva already in [34] and elaborated further by M. Daňková (cf. [43,46,48,52]. In 2006, Russian edition of the book by V. Novák, I. Perfilieva and J. Močkoř has been published in [63]. The book is quite successful in this vast market.

Another small group led by R. Bělohlávek (he left IRAFM OU in 2004) has been established in Olomouc. Let us refer especially to two books [45] and [54] where the fuzzy equational logic has been established. It has been shown in this book that when confining only to equalities, the generalized completeness in the sense of $\mathrm{Ev}_{Ł}$ can be obtained even without the assumption that the implication operation is continuous on $[0,1]$.

In 2005, two important directions in fuzzy logic have been opened. The first one was introduction of higher-order fuzzy logic by V. Novák in [58] — which is, in accordance with the classical type theory[6] is called *fuzzy type theory*. In principle, the $\lambda$-calculus has been extended and many-valued semantics has been introduced for it. The main motivation was to introduce a sound formal means for the development of FLb, that would be more convenient than $\mathrm{Ev}_{Ł}$ applied so far.

The second direction is *formal fuzzy mathematics*, the program introduced by P. Cintula and L. Běhounek in [60]. Their idea is to develop various areas of fuzzy mathematics formally using mathematical fuzzy logic in the style introduced by P. Hájek as the metamathematical frame. The mathematical background which is based on the class of fuzzy logics elaborated before is the *fuzzy class theory* [53]. In a sense, this is somewhat restricted higher-order fuzzy logic which might be equivalent with the fuzzy type theory above. At any case, everything that is formulated in the former can be easily formulated also in the latter when using the $\lambda$-calculus. Recall that the program of the formal fuzzy mathematics is a precise formulation of the program anticipated already in the book [15].

P. Cintula and L. Běhounek not only introduced the program of fuzzy mathematics but also established a group of people (T. Kroupa, R. Horčík, M. Daňková, C. Noguera and few other ones) who begun to realize it step by step. At any case, this is a significant attempt which can put fuzzy set theory to very well and strongly established grounds.

As for scientific meetings in the Czech Republic, in 2005 a small conference "The

[6]The type theory has been introduced already by B. Russel and A. Whitehead in their Principia Mathematica; later on more formally by A. Church in [1], and elaborated in detail by L. Henkin in [2,3] and P. Andrews in [44].

Logic of Soft Computing V" has been organized in Ostrava by IRAFM OU. The conference was the last but one of the series of this kind of conferences on formal fuzzy logic[7]. A special issue of some selected papers has been published in Fuzzy Sets and Systems 158(2007).

Another important meeting was the $5^{th}$ EUSFLAT Conference (European Society for Fuzzy Logic and Technology) which took place in Ostrava (Czech Republic) in September 11-14, 2007. The organizer of this conference was IRAFM OU. The conference was successful and the proceedings was published in [66]. In this conference, also the Working Group on Mathematical Fuzzy Logic (MathFuzzLog) of EUSFLAT was established. The main person in it is P. Cintula and it aims at conducting and promoting the research in Mathematical Fuzzy Logic.

The presentation of the results in fuzzy logic in the Czech Republic in 21st Century is not complete since there were very many activities of various kinds that would require much more space. We refer the readers to the WEB pages http:\\irafm.osu.cz\ and http:\\www.cs.cas.cz\ mathfuzzlog\ where more information including full references to many publications or to their preprints can be found.

## 6 An uncanny history of the GUHA method

In this section we will briefly mention another scientific idea developed in Czechoslovakia which is closely related to logic and whose history within science seems to be a mystery. Let us start with a question: "Who and when has founded the data-mining method and theory?". Those who say Agrawal, Srikant, etc. and refer to nineties are wrong. The founder of the idea of data-mining (without using this term which indeed has been introduced in 1990s) can be considered P. Hájek in cooperation with M. Chytil and I. Havel who published in 1966 the paper [4] introducing a formal theory of the method called GUHA (General Unary Hypotheses Automaton). This method was suggested to become a tool for exploration of new information from data containing boolean values[8]. The information has the form of "hypotheses" about relations among properties carried by the data. Note that boolean (or categorical) data occur quite often. Moreover, there is usually a straightforward way how other kind of the data can be transformed into boolean (categorical) ones. Let us also remark that one of the essential data-mining concepts introduced in nineties (cf. [23]) rediscovered some parts of the GUHA theory again under the name "associations".

The GUHA method is based on a special language of predicate logic extended by the concept of *generalized quantifier* (denoted by "$\sim$") which relates two formulas $A, B$, i.e. we deal with a quantified formula $A \sim B$. This formula characterizes a relation between properties represented by the formulas $A$ and $B$. The data take the role of a finite model in which a truth value of $A \sim B$ is determined using the four-fold table

[7] The first one was organized in Capri in 2001 and the last one in Malaga in 2006.

[8] The theory has quite soon been extended to data with categorical values.

| | $B$ | $\neg B$ |
|---|---|---|
| $A$ | $a$ | $b$ |
| $\neg A$ | $c$ | $d$ |

where $a$ is the number of objects having $A \,\&\, B$, $b$ is the number of those having $A \,\&\, \neg B$, $c$ of those having $\neg A \,\&\, B$, and $d$ of those having $\neg A \,\&\, \neg B$.

The basic considered quantifier is that of *simple symmetric association* saying that the formula $A \sim B$ is verified in the data (i.e. true in the given model) if $ad > bc$ (the number of coincidences is greater than that of non-coincidences). Another kind of quantifier is the *implicational one* saying that $A \Rightarrow B$ if $(a+b)/a \geq p$ for some reasonable number $p \in (0,1]$. There are wide classes of both kinds of quantifiers having various interesting properties (cf. [36, 47, 56]).

Quite soon, statistician T. Havránek[9] joined the group and developed several statistically based quantifiers belonging to the classes both of symmetric as well as of implicational ones. Since then, GUHA has been systematically developed until the beginning of eighties. Several other people joined the group, for example J. Rauch, who wrote the first computer realization, D. Pokorný, P. Kůrka, J. Ivánek, and few other ones. Regular seminars took place every year usually in a beautiful small village Alšovice in the north of Bohemia. In about 1978, an interesting new idea of GUHA80 was raised: to develop an automated knowledge discovery system. This idea was inspired by the new concept of *expert system* and the AM system (Automated Mathematician) developed by D. B. Lenat in his PhD thesis [6]. Unfortunately, the concept has not been concluded.

Many papers have been published mostly in Kybernetika and Int. Journal of Man-Machine studies in seventies and eighties. The most complete theory of GUHA can be found in the book [7]. In nineties, another branch of GUHA has been developed by J. Ivánek, B. Stejskal which has further been extended to automatic generation of expert system on the basis of data (see [14, 19][10]).

Let us stress that GUHA is very close to fuzzy logic. First of all, it is a logical method and can be taken as a realization of the theory of the, so called, *observational calculi* introduced in [7]. Second, the semantics of generalized quantifiers is quite artificially taken as boolean. Much more natural seems to take the number $p$ above (or the fraction $\frac{ad}{bc}$) as the *degree of truth* that the given hypothesis is valid. In fact, some steps in this direction have already been done (cf. [50]). Thus, the GUHA method can be more naturally formulated inside a special calculus of mathematical fuzzy logic. It also opens the door to generation of hypotheses just in natural language which seems to be very attractive for experts from various fields (cf. [59, 67]).

To conclude this brief section which would certainly deserve a more detailed elaboration, let us simply state that despite the fact, that theory of the GUHA method as well as many aspects concerning its realization have been widely published, still it happened that it has been almost forgotten. It seems that P. Hájek, T. Havránek and

[9] Unfortunately, he died in 1991 at the age of only 44.

[10] Remember the concept of GUHA80.

their cooperatives came with the idea of data-mining too early.

## 7 Conclusion

In this paper, we have briefly overviewed some of the main points in the development of fuzzy logic in Czechoslovakia and, since 1993, in the Czech Republic. Our presentation is far from complete. As we focused mainly on fuzzy logic and related topics we omitted a lot of results obtained in other areas of fuzzy mathematics such as fuzzy mathematical programming and other ones. There are also many results in fuzzy logic obtained only in this century that have not been mentioned because the paper would become too long. Therefore, we apologize to many Czech colleagues for not mentioning their work explicitly. Let us, at least, list some of the important omitted names: J. Ramík, M. Navara, J. Talašová, M. Pokorný, and few other ones.

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# The Gradual Arrival of Fuzziness in Switzerland

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**Abstract** Switzerland is renowned for many things: the mountains, Heidi, chocolate, bank accounts, the punctuality of public transport and, of course, (pretty expensive) high quality watches. However, Switzerland is probably most famous for exactness and accuracy, influencing Swiss mentalities and everyday life. Legend has it, that is to say, that in Switzerland you can set the watch by the trains – something unheard of in any other European country. Perhaps for this reason, in Switzerland it was especially difficult to take up concepts such as the fuzzy set theory developed by Lotfi Zadeh. Nevertheless, fuzzy set theory found its way into Swiss computer sciences, too, jolting the beliefs and convictions of Swiss scientists. The slightly belated gradual arrival of fuzziness in Switzerland is the topic of this article.

# 1 Introduction

Switzerland is renowned for many things: the mountains, Heidi, chocolate, bank accounts, the punctuality of public transport and, of course, (pretty expensive) high quality watches. However, Switzerland is probably most famous for exactness and accuracy, influencing Swiss mentalities and everyday life. Legend has it, that is to say, that in Switzerland you can set the watch by the trains – something unheard of in any other European country –, and that a minor delay of a local train can make it into the newspapers. Precision is the key to the sophisticated workmanship of the Swiss watchmaking industry, and accurate clocks are unquestionably vital to a reliable public transport system. To sum up: Attributes such as accuracy, exactness, and reliability are held in high esteem all over Swiss society and are supposed to be part of a particular Swiss national and cultural identity. This contributes to a profound uneasiness about things or ideas that cannot be detected by measurement or will not be described appropriately by the means of preciseness. Similarly, the scientific disciplines of the Swiss research community, especially the natural and social sciences as well as engineering and computing, tended (and still tends) to underestimate factors and phenomena that cannot be described adequately by anything else but terms and models relying on exactitude and unequivocalness. Of course, this does not apply only to Swiss researchers, but, apparently, in Switzerland it was especially difficult to take up concepts such as the fuzzy set theory developed by Lotfi Zadeh. As we all know, this theory takes into account a distinct logic of vagueness, inexactness, uncertainty and indetermination [Zadeh, 1965], which seems to be diametrically opposed to a quite static view of the world based on accuracy.

Nevertheless, fuzzy set theory found its way into Swiss computer sciences, too, jolting the beliefs and convictions of Swiss scientists – albeit rather late as in the most European countries the reception of Zadeh's ideas started much earlier [Seising & Arguelles Mendéz, 2015; Seising, 2005; Seising, 2009]. This slightly belated gradual arrival of fuzziness in Switzerland is the topic of this article. Based on the example of the Swiss Academy of Natural Sciences, the first part takes a look at the history of accuracy in Swiss research. The second part focuses on the fuzzy pioneers in Switzerland. Referring to the Swiss service industry, the third part elaborates on fuzzy marketing and management research, currently gaining traction in Switzerland. A brief conclusion completes this short history of the propagation of fuzzy set theory in Switzerland.

## 2 Accuracy and the Beginnings of the Swiss Natural Sciences

In 2015, the Swiss Academy of Natural Sciences, the oldest national scientific organization of the country, commemorated its 200-year anniversary [Kupper & Schär, 2015]. In 1815, the founding year of the Swiss Confederacy, cosmopolitan Swiss patriots and aficionados came together to establish this institution [Hupfer & Schär, 2015]. This was not the only academy of (natural) sciences that was founded in Europe between the first half of the 18th century and 1900 [Sellin, 2010]. This movement resulted from the emergence of the natural sciences that developed in the course of the Enlightenment from the traditional natural history that influenced the exploration of the world since Pliny's Naturalis Historiae [Jaeger, 2015]. These institutions, however, were not only supposed to house the new sciences, but also to further the nation's reputation and the building of a common civic identity – especially in the 19th century, the so-called Century of the Nation States.

This political, social and cultural background also explains why the first big project that was initiated by the Swiss Academy of Natural Sciences in 1820 was devoted to create exact geographical maps that were supposed to cover all parts of Switzerland [Hupfer & Schär, 2015; Kupper & Schär, 2015]. Here, too, the Swiss were not the only one as cartography – the topographical measuring of a partly still unknown world – was an exciting research field that attracted in many countries lots of men. To map Switzerland, very probably a triangulated mesh had to be formed. With the Topographic Maps of Switzerland – a series of 1:100 000 scale map, published from 1845 to 1864 – the young state was recorded comprehensively for the first time. Some years later, initiated by the International Geodesy Project, the Swiss Geodesic Commission reassessed the maps and found them insufficient. Therefore, the commission induced a revision resulting in very accurate three-dimensional models of Switzerland's topography [Hupfer & Schär, 2015]. Since, Swiss cartographers acquired a reputation for very good and precise maps. And perhaps it is no coincidence that important elements of the much frequented, granular-computing-based web-service Google Maps were developed by computer engineers in Switzerland – about 150 years after the first publishing of the Topographic Map of Switzerland [Allen, 2008].

The formation of a professional Swiss cartography since the 19th century is an interesting example illustrating the connection between accuracy and Swiss identity, right at the beginning of the history of the natural sciences in Switzerland as well as of Switzerland itself as a nation. Partly, this process is typical for the most European countries in the 19th century,

Social Sciences, the department conducts pure (i.e., information theory and mathematics) and applied research (i.e., design and engineering) in computer sciences and information systems. Unaware of Lotfi Zadeh's efforts in the 1960ies to found computer sciences as a discipline, the department of informatics at the University of Fribourg adapted his 'degrees of containment' in informatics by intuition [Zadeh, 1968].

At the beginning of the 1990ies, Jürg Kohlas, professor on theoretical computer sciences, and his colleagues at Fribourg took part in the European research project "Defeasible Reasoning and Uncertainty Management" [Kohlas, 1992a], founded as a collaboration of several European universities and other research institutions. This project was a large-scale research project looking into aspects of uncertainty in artificial intelligence [Besnard et al., 1993]. Thus, Kohlas and his fellow scholars became probably the first computer scientists in Switzerland doing research on fuzziness. Their work was the starting point and prepared the ground for a new generation of fuzzy researchers, not only at the University of Fribourg.

One of them is Kilian Stoffel, who – after postdoc-studies at the University of Maryland and the Johns Hopkins University in Baltimore – became full professor at the University of Neuchâtel in 1997. He firstly came into contact with fuzzy research at Fribourg where he obtained a PhD in information systems [Stoffel et al. 1993]. Recently, he pursues research on pragmatic applications of fuzziness, for example regarding to clustering or multi-criteria decision analysis [Stoffel et al., 2010; Albertetti et al., 2016]. Another protagonist, thereby, is the doctoral advisor of Stoffel; Béat Hirsbrunner established the "Pervasive and Artificial Intelligence Research Group" at the Faculty of Mathematics and Natural Sciences after his arrival at the University of Fribourg in 1986 [Kohlas, 2008].

However, the most important impact on the dissemination of fuzzy set theory as a field of research in Switzerland had the mathematician and computer scientist Andreas Meier who joined the Faculty of Economics and Social Sciences at the University of Fribourg in 1999. Next to other important projects in databases and web systems, Meier and his team introduced fuzzy marketing and management as a new research topic in Swiss information and computer sciences.

## 4 Fuzzy Marketing and Management Research: A new Perspective on Applied Fuzziness

In 2008 their activities in this field resulted in the foundation of FMsquare, a new research center on Fuzzy Marketing Methods at the University of Fribourg [Meier, Bambauer-Sachse & Donzé, 2009]. FMsquare is mainly

engaged in applying the notion of fuzzy sets to business and social challenges as in marketing and management. At the beginning, its main focus lied at marketing, but recently the center extended this perspective to management in general.

Today, data is said to be the new oil, but due to the growing information overload, it has become very difficult to analyze the huge amounts of data and to generate appropriate management decisions. This problem increases even more as lots of data are imprecise and include both quantitative and qualitative elements. Therefore, FMsquare attempts to extend traditional decision making processes by adding intuitive reasoning, human subjectivity and imprecision.

The FMsquare focus on innovative marketing and management issues is surely quite unique in the fuzzy community – not only in Switzerland but also internationally. Particularly for Switzerland, however, as an economy with a strong banking, insurance and service sector, such a research center appears key. So, to have best possible impact in society, as a consequence, the FMsquare research center zeros-in exactly on marketing and management challenges. To this end, in addition to research, the center works also closely with business and government (e.g., for building cognitive cities-to-be [Finger & Portmann, 2016]).

Several studies in this new research field were released since the foundation of the center. In 2008, preceding the founding of the Fmsquare within an inch, Nicolas Werro published a PhD-thesis dealing with fuzzy classification of online customers [Werro, 2008]. Other PhD-studies followed soon: In 2012, Edy Portmann developed a fuzzy grassroots ontology for online reputation management [Portmann, 2012], Michael Kaufmann an inductive fuzzy classification for marketing analytics [Kaufmann, 2012], and Daniel Fasel a fuzzy data warehousing method for performance measurement [Fasel, 2012]. In 2014, Luis Terán introduced a fuzzy-based recommender system architecture for smart participation [Terán, 2014]. Further doctoral theses are well on track already to be completed successfully over the next year.

In 2012, Meier and Portmann started together with Witold Pedrycz – professor at the University of Alberta, Canada, department of electrical and computer engineering, and former president to the North American Association for Fuzzy Information Processing and the International Fuzzy System Association – "Fuzzy Management Methods," an international book series, to establish a new publication medium for this emerging research field. In 2013, after spending his postdoc-studies at Berkley University (on invitation by Lotfi Zadeh), Edy Portmann accepted an assistant

professorship of information science at the University of Bern. The promotion of Swiss research on fuzziness is an important concern to him, wherefore he takes much interest in the intensification of the BeNeFri-research cooperation. Organized by Meier, Stoffel and Portmann, in late September 2016, the first International Conference on Fuzzy Management Methods (ICFMsquare 2016) took place in Fribourg. In the future, this conference is planned as an annual event with alternating locations in the BeNeFri-network. Additionally, this year also the FMsquare Foundation has been founded. This non-profit organization is dedicated to the application of fuzzy set theory on business and management [Stiftung FMsquare, 2016]. To conclude: With Fribourg as core, the BeNeFri-universities will continue to conduct humanistic (management) research based on fuzzy set theory to progress business, society and, thereby, all involved stakeholders.

## 5 Conclusion: Addressing Accuracy with the Principle of Incompatibility

This article gave an overview of the history of fuzzy research in Switzerland. The aim was to illustrate the history and the development of this promising research-field as well as its past, present and future contributors. Thereby, the shortness of this article does not only reflect the fact that this author is not an historian, but also that the history of fuzziness in Swiss computer sciences itself is still short indeed: Fuzzy research in Switzerland has only but started, and it remains a research-challenge that is worth to be pursued.
This article strives to do this, focusing on Swiss economy and society. Thereby the ‘principle of incompatibility’ [Zadeh, 1973] that proves itself as a corner stone of fuzzy sets theory is followed: It reveals the necessity of applying fuzziness to be able to understand and explain (social) reality. As fast as the social, cultural, political and economic complexities increase, the more precise categorical statements lose meaning and meaningful statements cease to be precise and categorical. Therefore, the principle plays a critical role in the promotion of fuzzy marketing and management methods in (accurate) Switzerland. Challenges of common concern have neither ultimate precisely defined answers nor unique scientific solutions. To deal with, Switzerland’s growing fuzzy community must develop an ability for a broad vision, encompassing large variety of different images, attitudes, and opinions. Then this research-challenge will certainly be successful.
A big thanks goes to Horst Bunke and Jürg Kohlas for their valuable information on the history of fuzziness in Switzerland. Both of them had, so to speak, a seat at the front-row and they readily shared their perceptions.

especially of those states that at that time only had started to build up a collective national identity of their own. In the case of Switzerland, however, the combination of accuracy and identity is particularly striking and persistent. Until today, accuracy, exactness, and reliability are supposed to be intertwined closely with "Swissness", a very adaptable, ambiguous term that is criticized frequently and harshly as 'mere' marketing phrase to sell Swiss products and services [Feige et al., 2010; Tanner, 2015]. Nonetheless, it is also part of Swiss self-perception (or at least self-display), and this may be one of the reasons why it is not be surprising that accuracy dominated Swiss scientific belief systems for more than 175 years after the establishment of the Swiss Academy of Natural Sciences.

But as it was said before: It took a little bit longer, but finally fuzzy set theory found its way into Swiss computer sciences. In the 1980ies and 1990ies researchers as Horst Bunke (University of Bern) and Jürg Kohlas (University of Fribourg) started to defy the paradigm of accuracy. Their dedication was the starting point for the Espace Mittelland region of Switzerland, encompassing the BeNeFri-Universities Berne, Fribourg, and Neuchâtel, to become the most prolific fuzzy region of Switzerland as it is today [Kohlas, 1992a, Kohlas, 1992b; Kohlas, 1992c].

## 3 Defying Accuracy: The Fuzzy-Pioneers in Switzerland

A pioneer of applied fuzziness in Switzerland is Horst Bunke. In 1979 he accomplished his doctorate, in 1985 the habilitation in computer science at the University of Erlangen (Germany), where he also was a member of the scientific staff, interrupted by postdoctoral studies at Purdue University (USA). In 1984, Bunke became professor of computer sciences at the University of Bern. There he was until his retirement in 2011 head of the research group "Computer Vision and Artificial Intelligence," with pattern recognition as his main research field [Riesen & Bunke 2015; Bunke & Riesen 2011]. In this context he also dealt with applied fuzziness [Dick et al. 2004; Schneider et al. 2001] and, especially, with fuzzy pattern recognition [Bunke & Kandel 2001; Kandel et al. 1997].

Some isolated research activities regarding the application of fuzzy set theory in the Swiss computer sciences took place at the Swiss Federal Institutes of Technology in Lausanne and Zürich (e.g., [Stoop et al, 2002]). However, the most fertile ground for fuzziness in Switzerland is the department of informatics at the University of Fribourg, founded in 1958 and thus the oldest university institute for information and computer sciences in Switzerland [Kohlas, 2008]. At the interface between the Faculty of Mathematics and Natural Sciences and the Faculty of Economics and

Finally, Astrid Habenstein helped me to interpret history, what needs to be appropriately owed at this place, too.




*Figure 1: Lotfi A. Zadeh and Edy Portmann at the IEEE Summer School on Semantic Computing 2010, University of California, Berkeley, Berkeley, USA*

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# On how Fuzzy Logic began in Spain (1974-1980)

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# 1 Introduction

With this paper I will try to cover but a part of what happened from 1974 up to around 1980. After this year everything acquired such a dynamics and density that I don't feel myself able to seriously write on it. For instance, at the end of XX Century the scientific production on Fuzzy Logic and related areas did grow in Spain in such a way that the figures on it shown this country in a fourth or fifth position in the world, when before 1974 such production was inexistent even if some people did have some news on fuzzy sets. Indeed, the first conference's papers appeared in 1977, and this was the year in which an explosion of fuzzy logic started in Spain; after 1980 many groups along the Spanish geography were active, with many Ph. D Dissertations presented or in course, and conferences and international journal papers published. In six years after the end of Franco's totalitarian regime, between 1975 and 1981, in general and with a very poor funding, scientific research in Spain did abruptly change towards an intellectual and socially challenging activity. This, besides considering that regular funding coming from the European Union, did not arrive to Spain before 1988, and after having joined the EU in 1986.

In the years covered by this paper, what in Spain was accomplished in fuzzy logic should be considered as coming from the personal effort made by the involved researchers, and just counting with minor amounts of funding from public institutions. For instance, my own work in the field of Probabilistic and Generalized Metric Spaces, from which many Ph. D. Dissertations and publications did arise began before 1966, but it was supported only between 1972 and 1977 by a private Spanish Foundation

# 2 The preliminary steps

**2.1**

Alfredo Deaño (1944-1978) was an Adjoint Professor of Philosophy at the University of Madrid (UCM, Universidad Complutense de Madrid), unfortunately passed away when he was just 33 years old, and because of a sudden heard attack. It happened very close to when he could have obtained a Chair of Philosophy.

Deaño was a young promise for Spanish Philosophy; granted with a serious classical philosophical background, he kept a great interest on modern linguistic philosophy, deeply knew the syntactical work of Chomsky, the then existing publications on Vagueness, and both the 1965 Zadeh's paper introducing fuzzy sets, and the 1975 Lackoff's (first and last 'fuzzy' paper) on linguistic hedges[2]. Deaño's passing away meant a deep

lost for the future of, at least, analytic philosophy in Spain. I never met him what, surely, was a pity for me and, perhaps probably, also in relation with a shared vision on representing natural language and commonsense reasoning that could have been influencing in the future.

In a time in which Spain boiled of democratic aspirations hold by young university professors, who devoted a lot of their time to activities trying to present modern tendencies in several aspects of culture, Deaño intensively participated with the direction of a very interesting collection of books, translated into Spanish some books, and did have nice public presentations concerning philosophy, and fuzzy sets in particular.

After his death, unfortunately and as far as I know, no Ph. D. Dissertation on fuzziness was presented in the area of philosophy up to the end of the eighties in last Century, when Alejandro Sobrino read under my advise his dissertation at the University of Santiago de Compostela. Curiously, Deaño and Sobrino are both born in Galicia.

In his 1974 book 'An Introduction to Formal Logic' (Introducción a la lógica formal)[3], Deaño not only devoted some pages to fuzzy sets and to linguistic hedges, but also nice and interesting comments concerning fuzzy logic in connection with natural language. Most philosophers did learn concepts of fuzzy logic, along the years in which this book was used as a textbook in the classes of logic for future philosophers.

**2.2**

Francisco Azorín, was a known and mature mathematical statistician whose main interest concerned the sampling problems in statistics, and that, being a full-professor at the UAM (Universidad Autónoma de Madrid) and a UNESCO international consultant, after 1977 held for some years the Honorary Presidency of the National Institute of Statistics (INE, Instituto Nacional de Estadística), after he had been developing intense works in South-America as a member of CEPAL, as well as being a professor at some universities in Venezuela and Chile. Son of a known architect that was a prominent socialist before the Spanish Civil War, he was a man of a great culture, and a member of the Spanish Royal Academy of Sciences since 1981. Worried by the too much precise questions posed to people in the official surveys and opinion polls, and by the precise answers required, after knowing the work of Zadeh, Azorin tried to convince the INE's statisticians for the need to pose imprecise questions, to accept imprecise answers and represent them by fuzzy sets. Just for this the INE published in 1979 his booklet 'Some Applications of Fuzzy Sets to Statistics' (in Spanish)[4], but and perhaps is in his discourse of entrance at the Royal Academy, 'Fuzzy Sets, Statistics, and Probability' (also in Spanish)[5], where he did show a

wide knowledge on fuzzy logic. Unfortunately, Azorín also passed away in the mid eighties by an unpredictable complication after a minor surgical operation.

Although concerning fuzzy logic the direct influence of Azorin was limited among professional statisticians, a little bit later it moved some academicians to consider imprecise events and fuzzy methodology. Among them, and impelled by his former advisor Sixto Ríos at UCM, Pedro Gil (Valladolid, 1947), a full-professor at the University of Oviedo, began to take into account fuzzy events at the mid eighties, and created a productive research group at that University. In it and very soon, it should be remarked his sister María Ángeles Gil (Valladolid, 1953), now also full-professor at the same University, and who did a stay of a couple of years in Berkeley with Zadeh. Anyway, all this goes further than 1980.

**2.3**

These two steps ahead done by Deaño and Azorín alone, should be considered partially unsuccessful but only concerning the true and further development of fuzzy logic in Spain, even if they greatly contributed to a good reception of fuzzy logic in this country. That the trials of Deaño and Azorín were unsuccessful for introducing fuzzy logic, does only refer to not having supervised young students on fuzzy logic in both the respective fields of Philosophy and Statistics. From this fact, their efforts did remain almost bounded to their own works that today, and without continuity, just bright as a personal, even if an excellent, endeavor that should be remembered as, at least, a sign of a time in which Spain was trying, basically by its most cultivated intellectuals, to abandon the cultural isolation in which the crazy and absurd totalitarian regime suffered between 1939 and 1975, did enclose the country. A regime that, not only for not preserving human rights, but also for the old religious and intellectual surveillance it pretended, cannot deserve any kind of a historical pardon under whatsoever point of view it could be seen.

## 3 A small jump

**3.1.**

In the summer of 1975, and one year after becoming a full-professor of mathematics at the Technical University of Catalonia (UPC, Universitat Politècnica de Catalunya), I read in a French newspaper an interview with Professor Arnold Kaufmann, in which his new book ‘Ensembles flous’was announced. It called my attention since, having been working on Probabilistic Metric Spaces for several years after 1966, I knew Karl Menger’s wide work and, in particular, his paper on hazy sets that he

Christianized by ‘ensembles flous’, since he published it in French and in the famous ‘Comptes Rendues’ of the Paris Academy of Sciences[7]. Nevertheless, Kaufmann’s book did not call too much my attention, since it seemed to me that its contents was fuzzy in itself; anyway, and mainly because of hazy sets, it lasted on me some curiosity moving me to search for the 1965 paper in which Lotfi A. Zadeh introduced ‘fuzzy sets’. Its reading abruptly changed my mind since I recognized in fuzzy sets not only a way of giving ‘extension’ to imprecise predicates, but also that they allow to mathematically represent the imprecise statements of natural language, and that in principle they were not related with probability as Menger’s hazy sets directly were.

It was for me a kind of a revelation that fuzzy sets appeared as a step ahead in the path going from Leibniz to Boole; for the first time, imprecise predicates P in a universe of discourse X, Q in a universe Y, etc., were represented by the membership functions of the fuzzy sets with linguistic labels P, Q, etc., and a composite statement linking elementary statements ‘x is P’, ‘y is Q’, etc., by connectives like ‘and’, ‘or’, and ‘not’, were also represented. This was clearly opening a window towards the representation of larger parts of natural language than classical sets, or Boolean algebras, allow to, and although no immediate axiom of specification does exist. It seemed to me that the famous ‘Calculemus!’ of Leibniz was approaching, and Zadeh’s ideas captured my attention for ever. Fuzzy sets appeared to me as context-dependent and purpose-driven; not only as a matter of degree as Zadek likes to say, but as a matter of careful design; it was a new concept that produced me a passionate interest that, from then, I always saw closer to that of Deaño than to Azorín’s one. I decided to work in the new field and started to search for the corresponding literature.

**3.2.**

Anyway, a few words concerning another reason for my sudden interest should be said. As a mathematician, I always kept a bad feeling towards simple generalizations, and hence my interest in fuzzy sets was not produced by the fact that they can be seen as a generalization of sets, that inside the set of all fuzzy sets in a universe X there is a subset isomorphic with the power set of X. On the contrary, and without denying such fact, what I did consider is that if in natural language there are more imprecise than precise words then, and at least from the point of view of mathematically representing language, it is more natural to look at sets as a kind of ‘degenerate’ fuzzy sets; this even if set theory, that is, mathematics, is necessary for building up such representation. A representation under which only classical sets do accept to be uniquely specified by an axiom like

that of Cantor-Zermelo and called of specification. That neither the membership function, nor the functions expressing the first connectives 'and', 'or', and 'not', are unique, make an important difference between Zadeh's fuzzy sets and classical sets. I did see fuzzy sets as the linguistic collective a predicate generates in the language, and not as a purely mathematical construct[8].

In this sense, I see mathematical models in fuzzy logic as something necessary, but not at all sufficient, for approaching the subtleties of natural language. From the beginning, I saw fuzzy logic as it appears in their last incarnation as Computing with Words[9], introduced by Zadeh a lot of years later at the end of XX Century. May be it is just an anecdote, but the fact is that, under the influence of the famous geometer Luis A. Santaló (Girona, 1911- Buenos Aires, 2001), professor at the University of Buenos Aires and Argentina's citizen by adoption after his exile from Spain because of the Civil War, I was not happy with the strong 'bourbakism' then prevailing at Spanish universities; Santaló was recognized as the world's leader of Integral Geometry. Hence and from my beginning as a researcher, I did search for topics with some relationship with reality, and I was fortunate by meeting them in the Probabilistic Metric Spaces of Menger, and later on the Fuzzy Logic of Zadeh.

## 4 The big jump

**4.1.**

Once I decided, at the end of 1975, trying to investigate on fuzzy sets, I started reading papers on the subject and very soon my attention was called up by the concept of 'fuzziness' that, not existing with sets, was new for me and show, specially, the important aspect of its measurability allowing to distinguish it from the philosophical term 'vagueness' with which it is again confused. The idea of fuzziness come to me from the 'indices of fuzziness' in the book of Arnold Kaufmann and, newly, it conducted me to a paper, the 1972 on 'non-probabilistic entropy' by Aldo de Luca, and Settimo Termini that made me realize that fuzziness can be measured and that, consequently, it is not a metaphysical concept. I certainly owe a big gratitude to Kaufmann's book since it impelled me to early know Zadeh and Termini papers, as well as to be later on and personally acquainted with both of them.

In 1976 the opportunity of organizing in Barcelona, and in July 1977, the 'First World Conference on Mathematics at the Service of Man', opened me the possibility of inviting Zadeh to deliver a main speech in it, as well as to opening to some Spaniards the opportunity of presentation of their first

papers on fuzzy logic. These people were Miguel Delgado, from the University of Granada, and me that, respectively, did contribute with papers on the concept of fuzzy-function, and on that of fuzzy entropy.

**4.2.**

Let me stop for a while in how Miguel Delgado was interested in fuzzy logic, since it reveals something that, from its very beginning, characterized this definitive step that, in particular, allowed Lotfi Zadeh to be acquainted with Spain, a country to which he returned many times in the following thirty years, and where he is both respected and loved. The second World Conference on Mathematics at the Service of Man was held in June, 1982 and at the Technical University of Las Palmas in the Canary Islands, and also counted with nice communications related with fuzzy logic. Nevertheless, it was already out of the period 1974-1980.

During the summers of 1975 and 1976 I did organize in the Technical University of Catalonia in Barcelona, a seminar called 'For the Initiation into Mathematical Research', to show new open tendencies to young people aiming at becoming researchers. These seminars were attended by many people, among which Miguel Delgado and his wife Amparo Vila from Granada, bright as some of the most enthusiastic attendants, and where both knew something on fuzzy logic. The idea for these seminars came to me from those organized in an American University and on which Karl Menger, lecturer in them, told me about. Such seminars resulted a good occasion to animate young researchers to start working in Generalized Metric Spaces[11], then my main line of research, and to let know fuzzy logic to mathematicians as a new and promising line of research in which many topics were deserving to be explored.

To disseminate new and open possibilities of research is, from then, a main characteristic constantly shared by those that start working in fuzzy logic from the last seventies of XX Century. In some years ahead it resulted in the establishment of several 'schools' of research that, under the leadership of some brave university professors, is what conducted Spain to a good international position first in fuzzy logic, and after in soft computing. In this way, the old type of a personal and isolated work earlier done by Deaño, and Azorín, was substituted by that of teams well intercommunicated with other teams around the world, and fuzzy logic benefited from many Ph. D. Dissertations written inside these teams, and its study did spread all around Spain as a prestigious subject of research.

# 5 Conclusions

**5.1.**

Many young doctoral students joined me in Barcelona, and Delgado in Granada. Simultaneously I entered in contact with people whom, in the 'Laboratoire d'Automatique et Analyse de Systemes (LAAS)' in Toulouse (France), and under Joseph Aguilar-Martin, were also interested in Fuzzy Logic, as it is Ramon López de Mántaras who, after doing in Toulouse his 'Thèse de Troisième Cycle' in which he used fuzzy sets, did a stay in Berkeley to study a Master and was working there under Zadeh. These contacts with the LAAS allowed me to be acquainted there with Michio Sugeno in the Fall of 1976, to know his 1974 Ph.D. Disssetation [12], and to invite him to visit both Barcelona and Granada in January 1977. Both Zadeh and Sugeno resulted, not only by their work but also personally, very influential in the future of fuzzy logic in Spain. Probably, we owe to the personal character of Zadeh the fact that our community of 'fuzzy researchers' always enjoyed an excellent level of friendship. It also deserves to be mentioned what, soon after 1980 up to today, and concerning areas related with fuzzy systems, Spanish researchers also owe to Claudio Moraga, professor at the Dortmund Technical University after his forced exile from Chile because of Pinochet 'coup d'etat', who not only through his work but also personally, influenced many of them in at least Barcelona, Granada, and Santiago de Compostela.

Finally, already in 1980, I published my first book on fuzzy sets, 'Conjuntos borrosos' (Fuzzy Sets)after publishing alone, in 1979, the paper that opened a line of research that I did continue for many years, the study of non-distributive forms of the connectives 'and' and 'or'. In this 1979 paper, whose title is 'Sobre funciones de negación para la teoría de conjuntos difusos' ('On Negation Functions for Fuzzy Set Theory'), and following a paper by Robert Lowen, I characterized all strong negation functions by means of solving a functional equation, a methodology I did learn when studying Probabilistic Metric Spaces and that, later on, with Claudi Alsina (who did his Ph. D. Disssertation on Probabilistic Metric Spaces under me and after being one year with Bert Schweizer in Ahmerst, Mass.) we did successfully apply to the study of the Aristotelian forms in fuzzy logic; up to the first decade of this Century, we jointly published more than seventy papers. The first of these series of papers devoted to connectives, and also co-authored with Alsina and me by Llorenç Valverde, was presented in 1980 in a conference in Barcelona with the title 'On non-distributive logical connectives for fuzzy set theory'[14]; in it the t-norms and t-conorms, we knew from our prior study of Probabilistic Metric Spaces,

were introduced in Fuzzy Logic. In that previous time, but already in 1981, I did introduce again in fuzzy logic a new concept also coming from Menger, that of T-Indistinguishability Operators [15], furtherly developed by some researchers the first of which were, among others, and not only in Spain, Llorenç Valverde and Joan Jacas in Barcelona. The Menger's trace in theoretic fuzzy logic is actually relevant, and I well remember what, in 1976 and in Chicago, Menger told me after regretting that Zadeh did not know on his hazy sets: 'Were Abraham Wald not passed away in the aircraft accident in India, today the subject would be an important one in mathematics'. In my view, it is a pity that up to today no 'famous' mathematician have been actually and deeply interested in Zadeh's ideas, and further than the more or less elemental mathematical models describing something of potential interest, and by taking into account that those ideas neither particularly refer to artificial languages, nor to formal deductive reasoning, but to the nuances of natural language and commonsense reasoning.

**5.2.**

After 1980, and as it has been said, people like (to quote but a few) the before cited Amparo Vila, and José Luis Verdegay in Granada, and in Barcelona people like Claudi Alsina, Francesc Esteva, Teresa Riera (who presented her Ph.D. Dissertation in 1981), and Llorenç Valverde (Ph. D. Dissertation in 1982), joined Delgado and me, and the way to future steps was definitively open.

In such a way, and between the first and the last eighties, it should be remarked the importance that meant the creation of two centers focalizing the research on fuzzy logic and related areas in Spain, and from which the research on fuzzy logic spread all around the country in different universities and departments. They were the Department of Computer Science and Artificial intelligence (DECSAI) at the University of Granada, and the Section of Artificial Intelligence at the Centre for Advanced Studies in Blanes (Spain), belonging to the CSIC (Spanish High Council for Scientific Research) that, later on, was converted in the today well known Institute for the Research in Artificial Intelligence (IIIA), placed close to Barcelona.

**5.3.**

In sum, if Deaño, Azorín and Trillas can be considered as those that are responsible of firstly introducing fuzzy logic in Spain, the unfortunate passing away of the two first caused that only the third could count with a possibility for going ahead. It was thanks to the job of people like Delgado, Vila, Verdegay, Alsina, Esteva, López de Mántaras, and many others. It is from all of them that the posterior work in fuzzy logic continued after several generations of researchers directly coming from those pioneers.

## 6 Acknowledgements

This paper is dedicated to those who most influenced my thought, in particular, to the late Karl Menger, and the late Luis A. Santaló. But, and especially to Lotfi A. Zadeh, and as a humble way of showing him my deepest acknowledgment for what I directly owe him through a friendship already lasting for almost forty years.

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# 50 Years of Fuzzy Sets in Germany

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## 1 Preface

Reviewing the development of fuzzy set theory in Germany poses some problems with respect to the focus of the review: Shall I focus on German authors, or on what happened in Germany during this time, or what contributions have originated in Germany? I shall focus on what happened in Germany in the fuzzy area within the last 50 years. A distinction between German and Not-German contributors to the fuzzy area becomes even more difficult because starting in the early 80s several colleagues and PhD candidates from other countries, such as China, Great Britain, India, Israel, Japan, USA spent some time in Aachen. Since there exist thousands of publications in this area, I will have to limit the references to a manageable number. That means I will only cite a number of characteristic German references and include also a few Non-Germans for purposes of calibration. This problem becomes even worse, because with the time Germans and Non-Germans team up as authors. Also events have happened in Germany and other events have been initiated in Germany, but happen elsewhere in the world. I shall include these two, but exclude even important events, that happen outside Germany or are not organized by Germans.

This paper presents a personal view. Hence, it will be from the point of view of somebody working in Operations Research (and not that of a mathematician or an economist). Finally, often the term "application" will be used. The reader should realize, that this a pretty fuzzy term. Many scientists imagine a problem, they model it formally and then apply a theory or method to it and they call it an application. By contrast to this, real problems have to be accepted as they are and then they can be modelled more or less appropriately. When I talk about applications I will mean "real" applications of the latter kind. For instance, the application of fuzzy set theory to linear programming is certainly an important application of a theory to a method but not a "real" application.

## 2 The Academic Press

As an Operations Researcher one aims to optimize processes and solve real applications, from the beginning of OR in interdisciplinary teams, because problems normally have features in several different disciplines, such as mathematics, engineering, psychology, law, business administration etc.. Working in OR I had always been frustrated about the fact, that in companies only problems on a very low level were considered to be solvable by OR, because problems on the level of top management were considered to be too ill structured to be accessible for OR. I was, therefore, very excited when I read Bellman & Zadeh's article [6] about decisions in ill-structured

environments in Management Science of 1970. I went to Berkeley right away and had fascinating discussions with Lotfi Zadeh. This started our research in Aachen in an interdisciplinary team (psychologists, mathematicians, statisticians, engineers, and business administrators). We had mathematical teams and empirical teams working in parallel primarily on two topics: 1. To fuzzify crisp OR methods (like linear programming etc.) and 2. How do real membership functions look like and do "min" or "product" model adequately the "and" humans are using. This research was particularly relevant for the so called "Expert Systems" which were very popular this time and very often not performing well. The research was mainly supported by the university in Aachen and by the German Science Foundation (DFG). This was the time when Kaufman published in France his first book on Fuzzy Sets [28] and Mamdani published the first results in the area of fuzzy control [36] and Yager published his first article decision making with several objective functions.

The results of our research were –concerning the first topic – fuzzy linear programming with one or more objective functions [70][72][21] and first applications of FLP [65][74]. Concerning topic 1 it turned out, that the min/max operators were not good models for the human "and" and "or" in general. Rather the "Hamacher Operator" and the family of Gamma-Operators modelled the human "and" and "or" acceptably well. [20][55][70][81][83].

At the end of our empirical research the DFG insisted that we show the applicability of our research and we started a project in which creditworthiness of bank customers – as estimated by bank employees- were modelled and empirically tested. It turned out, that the results of a fuzzy hierarchical aggregation of the determinants of creditworthiness were extremely good- and surprising to the bank managers [82][84]. However, it also turned out, that empirical research of the type we had been doing is very expensive. So, the DFG finished financing empirical research in the area of fuzzy sets.

In general research to fuzzify methods developed at this time in 4 directions: Optimization (FLP), knowledge-based-systems (expert systems and fuzzy control), classification, and pattern recognition.

Fuzzy Set research was not very popular in Germany at this time. In fact colleagues smiled at our research and considered it as "not enough scientific". Nevertheless, there were a few colleagues in other areas that were convinced of this approach and continued their research: Bock [10] worked and published in the area of fuzzy cluster analysis, Gottwald [18]did research in fuzzy sets on higher levels, Skala [49] worked on fuzzy logic and

Wechler [63] on fuzzy automata. With the exception of Aachen no courses on fuzzy topics were, however, offered at German universities.

Since researchers in the fuzzy area were distributed over the world and no common means of publication existed it seemed advisable to start one. So 1978 the Journal for Fuzzy Sets and Systems (FSS) was started and for the 21 years to come edited in Aachen. The first volume had approximately 600 pages. It is some indication of the importance of fuzzy set theory, that in the meantime the volume of this journal has grown to more than 3000 pages.

## 3 The Transition Phase

At the beginning of the 80s the interest in fuzzy sets started to rise slowly in Germany. The majority of publications concerned theory and methods. Fuzzy LP was further investigated [46], the first fuzzy programming language was offered [1][64], Fuzzy Petri nets were developed [35], and the first books in the fuzzy area were published by Germans [11][47][75][76]. Also the first publications on applications of fuzzy sets by German authors appeared: Inventory control [54]medical applications [2] and topics in social sciences were touched [73]. A real big application was the development of optimal routings for containerships by fuzzy LP [15]. FLP was used successfully in blending problems (paints, Champaign, rubber, etc.) Bezdek published his extremely important book on fuzzy clustering [7]. Also courses on fuzzy sets and expert systems were offered at the Technical University of Aachen.

At this time, it seemed to me that publications alone were not sufficient to advance the fuzzy area. Communications at that time were much slower than to day and researchers in the fuzzy area were located in different countries all over the world. I thought that talking to each other would be much more efficient. The result was, that IFSA (the International Fuzzy Systems Association) was founded in two steps in 1984: At first the European leading researchers met in Brussels and one month later I could ensemble interested academics in Hawaii in the framework of a NAFIPS conference and complete the founding of IFSA, which has proven very active since then. Already in 1985 there was the first IFSA conference in Mallorca (July 1 to 6, 1985) with 3 volumes of abstracts in conference proceedings.

During this time fuzzy control was already extensively used in Japan for consumer goods (video cameras, washing machines etc.) and even to direct a subway train in Sendai! In the German Economy one became aware of this, when a journalist, that had visited Japan, published an article in a German trade journal, warning the German enterprises, that they would lose

another market after the consumer electronics, if they would not do everything to catch up with Japan very fast. That article started in Germany the Fuzzy Boom.

## 4 The Fuzzy Boom in Germany

German scientists became interested in fuzzy sets and published books in this area [23][33] and also professionals published articles and books [57][61].

Hardly any trade journal related to computer science or computing dared not to publish articles on “Fuzzy Logik”. For instance: HighTech published an article with the headline “unleashed open-minded thinkers” (Entfesselte Querdenker) [17] and an interview with Lotfi Zadeh under the topic” Fundamentaler Wandel”. The front page of this issue presented a big picture of Zadeh with the slogan “How a new technique changes the world: Fuzzy Logic-the the magic formula for innovation”. Axon [29] reported under the topic “Queuing for Fuzzy Logic” on masses of visitors queuing in front of exhibitors of fuzzy products at the Interkama and the Hannover Industrial Fair 1972. Elektronik [34] included an interview of Lennemann who reported that “Germany is 12 month ahead of Japan”, see also [44] and [62]. Presented were mainly fuzzy control tools together with simple but impressive application examples, such as control of a travelling crane ( 7 fuzzy rules!), the checking of signatures, etc. These examples were mainly presented by big companies, such as Siemens, Omron, Klöckner-Moeller. By contrast to Japan, German companies hardly ever used “fuzzy” as an attractive label for consumer goods (such as cameras, washing machines, etc.). Probably because the German consumer attitude differs considerably from the Japanese.

A market research report in 1993 [3] lists 10 tools to develop fuzzy controllers of which 5 are German products. The same study one year later [4] lists already 10 fuzzy control tools, 3 control development systems and 4 Hybrid Neuro Fuzzy tools that can be used for fuzzy control, data analysis and pattern recognition. Of the last mentioned 2 were from Germany (DataEngine and FuzzyTech) and these were the only ones, that contained other operators than min/max. From the before mentioned 8 ware offered by German companies and institutions. More complete descriptions can be found in [12],[22],[51],[31] and [59]. Also Kiendl [30} published their first German book on Fuzzy Control.

Not only the number of “fuzzy shells”, i.e. tools to build fuzzy systems, was growing pretty fast at this time. Probably inspired by the large publicity of Fuzzy Sets, numerous areas not directly connected to control

engineering started applying fuzzy methods to their problems. I shall mention a few of them: De-icing of air planes. During winter days planes have to be de-iced in order to avoid icing problems during the flight. At airports their wings are sprayed with a liquid, the composition depends on temperatures. The blending is optimized by employees and they use a fuzzy tool to determine the optimal blending depending on the weather and other factors [32]. Other applications in the chemical industry can be found in [16][37] and [41]. About the use of fuzzy tools in robotics reports Palm in [43]. An example that has similarities with very modern developments is the "fuzzy car", that was already presented at the Hannover Fair in 1992 and raised then a lot of attention: A model racing car was equipped with sensors and a fuzzy controller which enabled the car to go fast and without any human interaction from point A to point B without touching any of the obstacles [60] placed between these two points. An interesting example in finance can be found in [5]. Biethahn [8][9] published two books on applications in business administration. In 1999 interesting contributions comparing Fuzzy Set Theory with Stochastics were published [53].

The development of fuzzy applications was also supported by an event that happened in 1992: During the decades preceding the fuzzy boom in Germany three areas that all imitated natural living had been developing apart from each other: Genetic algorithms and Evolutionary Programming, Artificial Neural Nets, and Fuzzy Set Theory. Theories in these areas complement each other [79]. In 1992 three congresses took place, in Japan, USA, and Germany (Aachen) at which these areas joined forces. The combination of these three areas was henceforth called "Computational Intelligence". The meaning of CI was even rigorously defined at the conference in the USA. The result was, that many tools and applications used methods of CI, i.e. combinations of methods from the above mentioned areas.

Applications and application directed research in the area of CI in Germany was not only undertaken by big companies but also supported by the state and by other institutions: The state of North Rhein Westfalia (NRW) spent many millions of Deutschmarks in the "Fuzzy Initiative NRW" [45] and enabled the start of new companies and institutions that concentrated on CI. As the best known one should probably mention the "Fuzzy Demonstrationszentrum Dortmund" (Dortmund), ELITE (European Laboratory of Intelligent Techniques)(Aachen) which is a private non-profit foundation, MIT GmbH (Management of Intelligent Techniques)(Aachen), and already existent but with a new division "Fuzzy Technology": Inform GmbH(Aachen).

To strengthen the cooperation between theory and practice ERUDIT (European Network in Uncertainty Techniques Developments for Use in Information Technology) was started in the mid-nintees by the European Commission DG III Industry-Esprit Program as a Network of Excellence which was run by ELITE in Aachen. It had established efficient communication in the area of CI between more than 250 European nodes. It aimed at stimulating innovations and advanced research and was encouraging trans-European training activities. ERUDIT was structured in six technical committees : Primary Industries; Process Industries; Components, Capital, and Consumer Goods; Traffic and Communication; Human-,Medical and Healthcare; Finance, Trade and Services. It would obviously exceed the scope of this article to describe even a large part of the projects of this network. In addition to the projects ERUDIT also organized the "European Symposium on Intelligent Techniques" once a year in different European locations.

Training in the area of CI had also increased very much in the meantime. More than 20 German universities offered courses in fuzzy sets. Several university chairs had begun with research in the area of CI. Conferences and information centers had started. The most important in Germany seem to me the "Dortmund Fuzzy Tage" (started 1991 and was once every year) ,the "Aachener Fuzzy Symposium" organized by Inform , which started in 1991 and occurred once a year, and the "Symposia for Applications of Fuzzy Technologies" organized by MIT in Aachen twice a year, which started 1992 and were organized twice a year. The largest event was probably EUFIT (European Congress on Fuzzy and Intelligent Technologies) which was organized by ELITE once a year and started in 1993. This congress, which had generally about 650 participants, had one special feature: Besides the normal presentation in various areas of applications of CI it provided space for about 25 to 30 exhibitors of real fuzzy applications, which, of course, always attracted a pretty large crowd of interested conference participants. This conference was organized until 1999.

## 5 After the Millennium

After 2000 the "Fuzzy Situation" in Germany changed pretty fast: Reports on fuzzy applications disappeared from the media. In particular trade journals and other application oriented media stopped publishing anything on either fuzzy techniques or fuzzy real applications. Obviously they had found other topics of which they assumed public interest. Since German producers of consumer goods (washing machines etc.) hardly ever

used “fuzzy” as an attractive label, one good also nowhere detect advertisements with the label “fuzzy”.

Public financial support ended for several reasons. That meant no more start-ups in this area. No more CI-conferences were organized in Germany. As far as I know no more university lectures were offered at German universities. I left the Technical University in Aachen in 1999 and there were no more fuzzy courses after this time.

The producers of fuzzy tools disappeared largely from visible market and those who were further active concentrated on narrow sectors of application. Inform and MIT in Aachen, for instance, which had used their tools (DataEngine and fuzzyTech) in very many areas of application, focused either on customer segmentation in banks or on fraud detection and prevention in financial services. That, of course, might be due to the fact, that the possession of a fuzzy tool alone does not mean successful operations. It needs marketing activities to become known by potential customers.

1973 a survey [45] came to the conclusion, that 17% of the companies used already fuzzy technology and that 43% of the participants of the survey knew about fuzzy technology. This is certainly no longer the case in Germany. Nowadays some still remember the word “fuzzy” but know of very few companies that still use this technology.

More scientifically oriented journals, such as the Journal for Fuzzy Sets and Systems, still published articles by German authors [27][39][40][42][48][56], but they were all pretty mathematically oriented and the relative portion of German authors declined compared to authors from other countries. An exception is the year 2015 when more German scientists contributed to the two special issues of FSS and Informatik Spektrum dedicated to 50 years of fuzzy set theory [13][14][19][24][26][52][66].

With the exception of the two issues of FSS[24][52][66] and Informatik Spektrum [13][14][19][66] dedicated to 50 years of Fuzzy Set Theory only few German contributions can be found during the years after 2000.

It should also be realized, that soft-and hardware has become considerably faster since the 90s of last century. Hence, the need for systems that provide solutions in very short times has reduced in many areas. That is probably also one of the reasons that expert systems have lost very much of their attractiveness. Also the influence of computer science or digital computing in general has risen considerably since the 90s of last century. That has strengthened in general the digital thinking and, hence, made it more difficult for a theory like fuzzy set theory to be understood and

accepted, particularly if it is taught nowhere. A little personal story at this point: When my son was 11 (in the 90s) and had set theory at school, he was very interested in what I was doing, i.e. in fuzzy sets. One day he asked his math teachers what he knew about fuzzy sets. The answer was: "That is nonsense, an element can only belong to a set or not." When my son answered, that he somewhat disappointed because there were already hundreds of publications on the theory of fuzzy sets, the relationship between him and his teacher had been spoiled for a very long time.

There may also be another reason for the present situation of fuzzy sets in Germany: Fuzzy set theory has been sold for a very long time as a theory for uncertainty modelling and as a competitor to probability theory and statistics. In the meantime, there exist more than twenty uncertainty theories [78] and the contexts in which they should be used have not been defined clearly enough to allow the practitioner to decide which one to use when. So in practice very often the "Hammer Principle" is being applied: If I have a hammer in my hand everything starts looking like a nail. In this case the "hammer" is certainly probability theory.

## 6 Conclusions

Research in the fuzzy area in Germany seems to be limited to mathematical developments by those researchers who have already been working in fuzzy sets in the last century. German authors still publish in scientific journals, but relative to foreign researchers their number seem to decrease. Obviously there is still ample space for mathematical research, but Germany will in the future not one of the leading countries. The problems seems to me, that in this type of research it is generally assumed that numerical expressions are on an absolute scale level, which cannot be reached for numbers estimated by humans. As is known in psychology and probably also in linguistics human estimates generally only reach an ordinal scale. This can also not be cured by using membership functions of degree 2, because then the numerical effort becomes excessive.

Empirical research (on real shapes of membership functions or on context dependent operators) at the present time in Germany is not known to me. This, however, would be the bridge between formal theory and models of human behavior and reasoning. Also education or training in the area of fuzzy sets in Germany is not known to me. On the contrary for reasons already mentioned above, digital thinking seems to become more and more common.

Fuzzy tools and techniques do still exist. fuzzyTech, Riskshield and DataEngine, for instance, are still being used but in rather narrow areas.

Many tools, however, provided only the min/max theory and triangular membership functions. It had been attractive to use such simple models for simple problems, but more complex problems needed more powerful approaches. Fuzzy Linear programming is even available as 0/1 version [80], but it is very difficult to convince somebody to make use of it. The same is true for many modern heuristics.

These developments have reduced real applications of fuzzy sets in Germany drastically. Even though I am convinced, that the potential for using these methods is still very large. In all applications in which thoughts, texts, or communications have to be semantically correct translated into formal models to be processed on computers or to maintain the real meaning (for instance in laws !) fuzzy sets will also in the future be a very valuable tool.

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