Archives for the Philosophy and History of Soft Computing
Issue 1/2018
INTERNATIONAL ONLINE JOURNAL OPEN ACCESS
ISSN 2341-0183

Interview with Prof. Dr. Erich Peter Klement

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Figure 1: Erich-Peter Klement

Rudolf Seising: Peter, you are from Austria and it seems that Austrian researchers have a deep love for logical thinking. There was Wittgenstein and there was Popper. There was the Vienna Circle in the first decades of the 20th century. What do you think about that? Do you see yourself in this tradition?

Peter Klement: It is true that, in the first third of the 20th century, Vienna was one of the most flourishing and influential cities of the world, not only with respect to music, literature, architecture, and the fine arts, but also in science where the Vienna Circle played a major role on an interdisciplinary level. You mentioned the philosophers Ludwig Wittgenstein[1] and Karl Popper,[2] but there were also other philosophers, mathematicians, physicists and logicians like Hans Hahn,[3] Moritz Schlick,[4] Rudolf Carnap,[5] Kurt Gödel,[6] Karl Menger,[7] Richard von Mises,[8] Otto Neurath[9] and, to a smaller extent, Alfred Tarski,[10] the economist Oskar

[1] Ludwig Josef Johann Wittgenstein (1889-1951) was an Austrian philosopher. His important works were the firstly in German published *Logisch-philosophische Abhandlung* (*Tractatus logico-philosophicus* 1921) and the postum published *Philosophische Untersuchungen* (Philosophical Investigations, 1953).
[2] Sir Karl Raimund Popper (1902-1994) was an Austrian philosopher. His work founded the so-called Critical Rationalism. One of his important works was the *Logik der Forschung* (*The Logic of Scientific Discovery*, 1935).
[3] Hans Hahn (1879-1934) was an Austrian mathematician and philosopher.
[4] Friedrich Albert Moritz Schlick (1882-1936) was a German physicist and philosopher, and one of the founders of the Vienna Circle.
[5] Paul Rudolf Carnap (1891-1970) was a German philosopher and a leading member of the Vienna Circle. His important works are *Der logische Aufbau der Welt (The Logical Structure of the World. Pseudoproblems in Philosophy*, 1928) and *Logische Syntax der Sprache* (*The Logical Syntax of Language,* 1934).
[6] Kurt Friedrich Gödel (1906-1978) was an Austrian logician, mathematician, and philosopher. He published his two incompleteness theorems in 1931. Later he immigrated to the US.
[7] Karl Menger (1902-1985) was an Austrian mathematician. He was a member of the Vienna Circle and immigrated to the US.
[8] Richard Edler von Mises (1883-1953) was an Austrian mathematician. He was a member of the Vienna Circle. Later he immigrated to Turkey and then to the US.
[9] Otto Neurath (1882-1945) was an Austrian economist, historian and philosopher. He was a member of the Vienna Circle. Later he immigrated to Turkey and then to Great Britain.
[10] Alfred Tarski, originally Alfred Tajtelbaum (1901-1983) was a Polish mathematician and logician. He was a member of the Lwów–Warsaw school of logic and the Warsaw school of mathematics. Later he immigrated to the US.

Morgenstern,[11] and the jurist Hans Kelsen[12] (author of the Constitution of the Austrian Republic, 1920).

Looking at their biographies, you see that (with the exception of Hans Hahn who died in 1934 and Moritz Schlick who was assassinated in 1936 by a former student), all the others left the Austrian "Ständestaat" and the German "Third Reich", mostly for the United Kingdom and the United States, and none of them returned to Austria after the end of World War II – in general, they were not even invited or asked to do so. So it would be an exaggeration to say that, in high school or as a young student, I was influenced in a significant way by these great scientists.

Only when already working in the field, I realized the important and visionary contributions of Kurt Gödel (whose Incompleteness Theorem from 1931 was one of greatest mathematical achievements of the century, but who also contributed significantly to, e.g., intuitionistic logic) and Karl Menger (who in 1942 coined the terms "triangular norms" and "statistical metric spaces") for the development of fuzzy set theory.



Figure 2: Berthold Schweizer (1929–2010)

[11] Oskar Morgenstern (1902-1977) was a German economist. He got his PhD in political science from the University of Vienna. At the "Anschluß" of Austria to Nazi-Germany, he was in the US and he decided to remain there. He and John von Neumann (1903-1957) wrote *Theory of Games and Economic Behavior*, (1944).

[12] Hans Kelsen (1881-1973) was an Austrian jurist, and philosopher. He immigrated to the US in 1940.

And one of my favourite quotations, describing in such an accurate way the intentions Lotfi Zadeh had when creating fuzzy sets, can be found in Karl Popper's "In Search of a Better World":

'Clarity is an intellectual value in itself; exactness and precision, however, are not. Absolute precision is unattainable; and there is no point in trying to be more precise than our problem demands.'

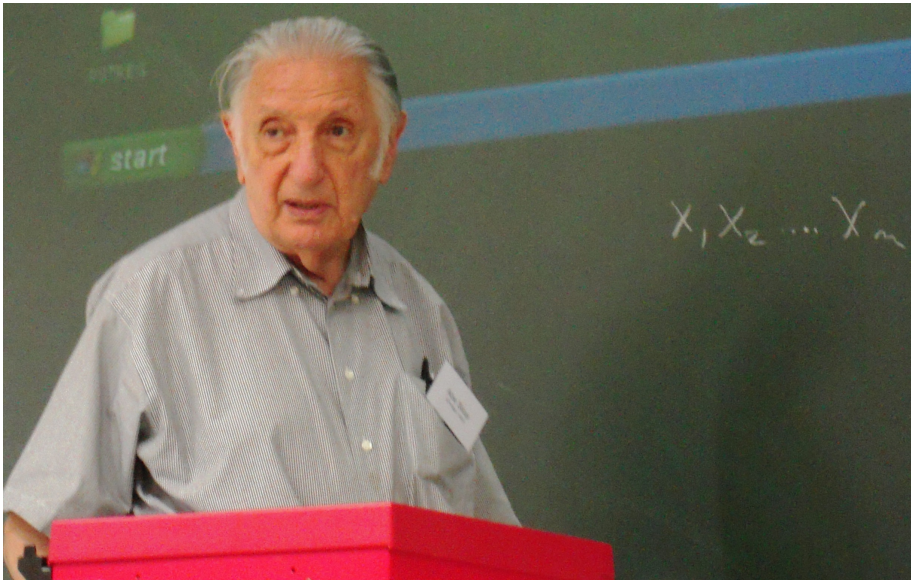


Figure 3: Abe Sklar (born 1925)

Rudolf Seising: What was the field you were working in research first and why did you realize Gödel, Menger and all the other scientists and philosophers that you mentioned? Were you interested in history or philosophy of science already in the high school at the university? Where did you start studies, which disciplines did you study and what was the subject of your masters' and Ph. D. thesis?

Peter Klement: I started my studies in mathematics and physics in 1967 at the University of Innsbruck, and my main interest was in measure theory and integration. I obtained my doctorate after four years at the age of 22 with a thesis in integration theory (no intermediate master was required at that time). After my change at the Johannes Kepler University in Linz I

soon got a chance to teach courses in point-set topology, while our group in Linz started to do research in probability theory (mainly in point processes). In 1976 my professor suggested me to read Ulrich Höhle's paper "Maße auf unscharfen Mengen" [1].[13] This was my first encounter with the idea of fuzzy sets, I became fascinated by the elegance and simplicity of Lotfi A. Zadeh's [2] approach,[14] and I tried to read as much as possible from the existing literature in the field (which was still comparably small: a 1975 "Bibliography on fuzzy sets and their applications" in [3, pp. 477–496] listed a total of 238 items). In that time also my interest in logic, many-valued logic, foundations of probability, and in the philosophy of science started to grow, especially when I became aware of some early traces of fuzzy sets in, e.g., Karl Menger's paper on statistical metrics [4] (where the concept of triangular norms was presented for the first time), but also in the work of Kurt Gödel.

Rudolf Seising: So, I see that you became interested in Menger's work on statistical metrics and similar subjects like probabilistic geometry [5] more than 20 years after he had written these papers. Have you ever met Menger or Bert Schweizer[15] or Abe Sklar[16] who had collaborated with Menger after he immigrated into the US?

Peter Klement: Indeed, Karl Menger not only coined the term "triangular norms" (because of their role for the validity of the so-called triangle inequality in "statistical metric spaces" [4] (which were called "probabilistic metric spaces" in [6]), but he also introduced the notion of "ensembles flous" [7] ("hazy sets" in English [8]) with a probabilistic

[13] Ulrich Höhle (born 1943) is a German mathematician and he was professor in the department C – Mathematical and Natural Sciences at the University of Wuppertal, Germany.

[14] Lotfi A. Zadeh (1921-2017) was a professor of electrical engineering the University of California, Berkeley. In 1964 he founded the theory of fuzzy sets. For more details see: Seising, Rudolf: *The Fuzzification of Systems. The Genesis of Fuzzy Set Theory and Its Initial Application– Developments up to the 1970s* (Studies in Fuzziness and Soft Computing, Vol. 216) Berlin, Heidelberg [et al.]: Springer-Verlag 2007.

[15] Berthold Schweizer (1929–2010) was a German mathematician. His family emigrated into the USA. At the Illinois Institute of Technology in Chicago he got his Ph.D. in 1956 under supervision of Karl Menger. Together with Abe Sklar he developed the theory of probabilistic metric spaces, that was conceptualized by Karl Menger in 1942.

[16] Abe Sklar (born 1925) is a US- mathematician and professor of applied mathematics at the Illinois Institute of Technology (IIT) in Chicago.

interpretation – in contrast to classical sets he called "rigid" ("ensembles rigides"). In his paper on Karl Menger [9], Seymour Kass[17] claims that '... hazy sets were rediscovered and renamed "fuzzy sets" in' [10]. It is interesting that the term "ensemble flou" was also used in 1968, without being aware of Zadeh's work [2], by the French linguist Yves Gentilhomme [11].[18] A very informative source for the pre-history and early developments of fuzzy sets (mainly, but not exclusively from a French perspective) is the excellent paper by D. Dubois[19] and H. Prade[20] [12].

Remarkably enough, in several papers [4,7,8] a strong motivation for Menger's ideas came from the so-called "Poincaré paradox"[21] described in 1905 [13]: in a physical continuum, the equality (or, more precisely, indistinguishability) of objects may not be transitive. This means that two objects *A* and *B*, which are both indistinguishable from an object *C*, are not necessarily indistinguishable. An early attempt to deal with this paradox by means of some equivalence relation based on a triangular norm was made by Ulrich Höhle [14].

Unfortunately I never had the opportunity to meet Karl Menger in person, but I was in touch with Bert Schweizer already in 1980 when I visited him and Hung T. Nguyen at the University of Massachusetts at Amherst. Later on, Bert wrote an introductory chapter "Triangular norms, looking back – triangle functions, looking ahead" to the book [15] Radko Mesiar[22] and myself edited as a follow-up of the 24th Linz Seminar on Fuzzy Set Theory (2003) "Triangular Norms and Related Operators in Many-Valued Logics". A very special opportunity was the wonderful conference

[17] Seymour Kass (1926-2013) was a US-mathematician and professor at the University of Massachusetts, Boston.

[18] Yves Gentilhomme (1920-2016) was a French linguist.

[19] Didier Dubois (born 1952) is CNRS Research Advisor / Directeur de recherche CNRS, Institut de Recherche at the Informatique de Toulouse (IRIT), CNRS and Université Paul Sabatier, Toulouse, France.

[20] Henri Prade (born 1953) is CNRS Research Advisor / Directeur de recherche CNRS, Institut de Recherche en Informatique de Toulouse (IRIT), CNRS and Universit´e Paul Sabatier, Toulouse, France.

[21] Jules Henri Poincaré (1854-1912) was a French mathematician, theoretical physicist, engineer, and philosopher of science. His most important works are *The Foundations of Science* (1921), *Science and Hypothesis* (1902), *The Value of Science* (1905), *Science and Method* (1908).

[22] Radko Mesiar (born 1951) is professor in the Faculty of Civil Engineering, Department of Mathematics and Descriptive Geometry at the Slovak University of Technology, Bratislava, Slovakia.

"Copulæ: the 50th Anniversary" organized by Carlo Sempi[23] in Lecce (2009): we met Bert Schweizer and Abe Sklar (and a number of other copula researchers) in a beautiful place, heard what they had to say and enjoyed the social program, not knowing that this should be Bert's last trip to Europe ...



Figure 4: Carlo Sempi, Bert Schweizer, Abe Sklar and Peter Klement at Sempi's home in 2009.

Rudolf Seising: I think that Menger's work came close to the mathematical structure of the theory of fuzzy sets but concerning the meaning behind that he did not envisage something that differs from probabilities – as Lotfi Zadeh did when he founded the theory of fuzzy sets. I think that Menger misunderstood Zadeh's writings when he read one of the first papers introducing fuzzy sets and then he compared this with his approach in 1966 as follows: "In a slightly different terminology, this idea was recently expressed by Bellman[24], Kalaba[25] and Zadeh under the name

[23] Carlo Sempi (born 1948) is professor in the Department of Mathematics and Physics at the University of Salento, Lecce, Italy.
[24] Richard Ernest Bellman (1920-1984) was a US-mathematician. He was professor of mathematics, electrical engineering and medicine at the University of Southern California. In 1953 he introduced dynamic programming.

fuzzy set (These authors speak of the degree rather the probability of an element belonging to a set)" [10, 16]. In my view, Menger did not see that this "slight difference" between "degrees" and "probabilities" is not only a difference in terminology but in the concept's meanings.

When I was with the *European Centre for Soft Computing* in Mieres (Asturias), Spain, in Enric Trillas' office there was a photograph of Menger and Trillas.[26] This must have been taken in the 1980's. Is there any relationship between your work on t-norms and t-conorms and the works in fuzzy mathematics by Trillas and other Spanish mathematicians?

Peter Klement: For many mathematicians and statisticians it is tempting to interpret any number between 0 and 1 as the probability of some event, and there are also different concepts of probability (see, for instance, the paper by Rudolf Carnap [17]). In the early eighties, the English Bayesianist Dennis V. Lindley[27] published a paper in the International Statistical Review [18], claiming that probability theory was able to model all types of uncertainty. This initiated an intensive discussion which included also contributions by Glenn Shafer[28] and Lotfi Zadeh. And more recently, in his book "Understanding Uncertainty" [19, chapter 5, p. 71], Lindley declared 'Whatever way uncertainty is approached, probability is the only sound way to think about it.'

For me, fuzziness has to do a lot with the imprecision and vagueness of our natural languages, a phenomenon where I don't know of any random experiment or polling or betting scenario to describe it properly. Zadeh himself later on described it by the term 'computing with words' [20].

Having met Ramon Lopez de Mantaras[29] already in 1979 during our joint time in Berkeley, I got in touch quite early with Enric Trillas (who

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[25] Robert E. Kalaba (1926-2004) was a professor of biomedical engineering, electrical engineering and economics at the University of Southern California.

[26] Enric Trillas Ruiz (born 1940) was professor at the universities of Barcelona, Technical of Catalonia, Technical of Madrid. He was an Emeritus Researcher at the European Center for Soft Computing (ECSC). He is an Honorary Emeritus Professor at the University of Oviedo.

[27] Dennis Victor Lindley (1923-2013) was an English statistician. He was an advocate of Bayesian statistics, and he had great expertise in decision theory.

[28] Glenn Shafer (born 1946) is a US-mathematician and statistician. With Arthur P. Dempster he co-authored contributions to the Dempster-Shafer Theory. He is a University Professor at Rutgers University.

[29] Ramon Lopez de Mantaras (born 1952) is Research Professor of the Spanish National Research Council (CSIC) and Director of the Artificial Intelligence Research Institute (IIIA).

considers himself as one of the last students of Karl Menger). I visited him, Claudi Alsina[30] and Ramon in 1981 in Barcelona for one week. We had long discussions, particularly about triangular norms and their use in fuzzy set theory, which had a deep impact on my work, and we stayed in close contact all over the years. As an example, Enric Trillas was an early participant in my "Linz Seminar on Fuzzy Set Theory" in 1983, where he talked about "An approach to the meaning of fuzziness".

Rudolf Seising: Can you give us some remarks on the history of the series of your "Linz Seminar"? – As far as I know these seminars have been visited by many fuzzy mathematicians, logicians and other researchers in the last 25 years. When and why did you initiate this series?

Peter Klement: The idea came up in September 1978 during a walk at the Vieux Port in Marseille: Robert Lowen[31] and myself attended a conference with many participants, parallel sessions, little time for the talks, and almost no time for discussions because of a very tight schedule. We designed a real contrast program: few participants living together in a rather secluded place for a week, and a lot of time for scientific interaction between them. I managed to get some little financial support from our ministery, and in September 1979 we started the first "Linz Seminar on Fuzzy Set Theory" with eight participants from Belgium, France, Germany, the United States and Austria, among them Ulrich Höhle and Henri Prade. Everybody gave a one hour talk on the first day, and the remaining four days we had discussions about fuzzy numbers, random sets, t-norms etc. The presentations were typed on our typewriters, copied at my university and bound in a primitive way – and most of them were reviewed in the "*Mathematical Reviews*" (now "*MATHSCINET*") and "*Zentralblatt*" (now "*zbMATH*").The original plan was that Bob Lowen, Ulrich Höhle and myself would organize the following seminars, according to the rotation principle, in Belgium, Germany and Austria, but for some reasons Linz continued to be the place of choice. Until 1987 the "Linz Seminar" was organized jointly by the three founders, then a permanent Program Committee was installed, and each seminar was devoted to a more specific topic. A special feature was also the invitation of young, promising researchers (after the fall of the Iron Curtain many of them coming from the Eastern part of Central Europe who had little contact to the scientific community before), and also the

[30] Claudi Alsina (born 1952) is professor of mathematics at the Universitat Politécnica de Catalunya in Barcelona, Spain.

[31] Robert Lowen is professor of mathematics at the University of Antwerp, Belgium.

invitation of distinguished mathematicians who had not yet worked in fuzzy logics and fuzzy set theory. Over the years, so far 37 editions of this seminar were held. The structure has changed partly: there are more participants now (up to 40) and, subsequently, more presentations. But still there are no parallel sessions, the location is always the same (Bildungszentrum St. Magdalena), and we have a full week of stimulating discussions and fruitful cooperation. Many research papers emerged from this series of conferences, some of them were published in five edited volumes and in a dozen of special issues of *Fuzzy Sets and Systems* and *Quaestiones Mathematicae*, were we presented selected and extended contributions to the seminars in a printed form.



Figure 5: Cover of the Proceedings to the International Seminar on Fuzzy Set Theory, Linz, Austria,



INTERNATIONAL SEMINAR ON FUZZY SET THEORY

JOHANNES KEPLER UNIVERSITÄT LINZ SEPTEMBER 24-29, 1979

Program for Monday, September 24

9:00 a.m.	Henri M. Prade Nomenclature of fuzzy measures
10:00 a.m.	Erich Peter Klement Extension of probability measures to fuzzy measures and their characterization
11:00 a.m.	Werner Schwyhla Conditions for a fuzzy probability measure to be an integral
12:00 noon	Ulrich Höhle Upper semicontinuous fuzzy sets and their applications
13:00 p.m.	Lunch
2:30 p.m.	Josette & Jean-Louis Coulon Fuzzy boolean algebras
3:30 p.m.	Richard H. Warren Fuzzy topological properties
4:30 p.m.	Robert Lowen Compact Hausdorff fuzzy topological spaces are topological

Figure 6: Program of the first day of the International Seminar on Fuzzy Set Theory, Linz, Austria, September 24-29, 1979.



Figure 7: H. W. Martin, Didier Dubois, Robert Lowen, Ronald R. Yager, Ulrich Höhle, Erich Peter Klement. Photo by W. Schwyla. 2nd International Seminar on Fuzzy Set Theory, Linz, Sept. 1980.



Figure 8: At the International Congress on Applied Systems Research & Cybernetics in Acapulco, Dec. 12-15, 1980: G. Jumarie, Henri Prade, Masao Mukaidono, Ronald R. Yager, Robert Féron, Lotfi A. Zadeh, Erich P. Klement, Dan Ralescu, W. H. Benson.

Rudolf Seising: Is it right to say that the participants of the Linz Seminars were principally involved in mathematical aspects of the theory of Fuzzy set, e.g. algebraic structures, t-norms, t-conorms, fuzzy measures, etc.? I think that in that case there were many disputes about the direction of the theory of fuzzy sets, e.g. about type-2 fuzzy sets and intuitionistic fuzzy sets. Would you please narrate retroperspectively about these debates and discussions?

Peter Klement: In the early years, we were busy with the concept of fuzzy numbers and with the use of algebraic operations such as t-norms in fuzzy set theory. It became increasingly clear that fuzzy sets and fuzzy logics are closely interrelated [21], leading to an extensive study of MV-algebras, Heyting algebras and P. Hájek's BL-algebras [22].[32] Also, the unit interval was quickly replaced by much more general structures, as first suggested by J.A. Goguen [23,24].[33] In most cases, residuated lattices were considered, and subsets of the unit square or the unit cube were rarely investigated. Also, type-2 fuzzy sets were not a "hot topic" at the "Linz Seminars", with the notable exception of the work of Carol[34] and Elbert Walker[35] who studied the algebra of the corresponding truth values [25].

The fact that K.T. Atanassov[36] [26,27] called his concept, where he considers both a degree of membership and a degree of non-membership, "intuitionistic" fuzzy sets because of the absence of the *Law of Excluded Middle*, led to a major controversy. It was noted that this generalization had nothing to do with the idea of intuitionism in the beginning of the 20th century going back to L.E.J. Brouwer [28].[37] Moreover, The Law of Excluded Middle did not hold in Zadeh's original proposal [2], and a double negation used for "intuitionistic" fuzzy sets leads to a contradiction in the original

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[32] Petr Hájek (1940-2016) was a Czech mathematician and logician. With *Pavel Pudlák he published Metamatemathics of First-Order Arithmetic (Berlin: Springer 1993).*

[33] Joseph Amadee Goguen (1941-2006) was a US computer scientist and professor at the University of California and Oxford University. He was also a researcher with IBM and SRI International. He earned his PhD in mathematics from the University of California, Berkeley in 1968, where he was a student of Lotfi Zadeh.

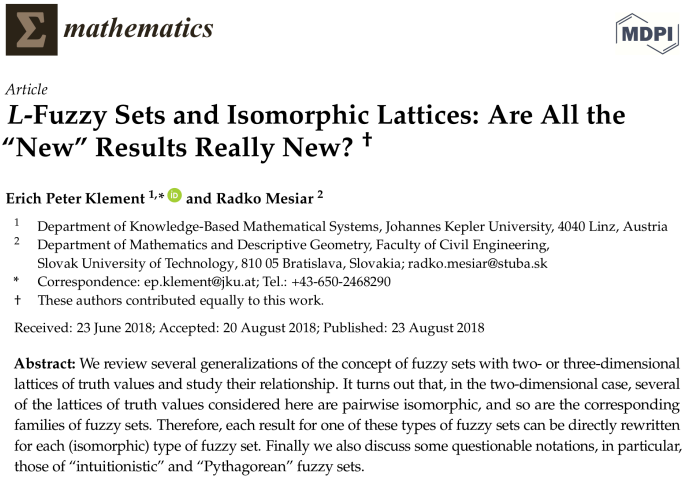
[34] Carol l:Walker is a professor of mathematics at the New Mexico State University in Las Cruces, New Mexico, US.

[35] Ebert A. Walker (1930-2018) was a professor of mathematics at the New Mexico State University in Las Cruces, New Mexico, US.

[36] Krassimir Todorov Atanassov (born 1954) is a Bulgarian mathematician. He introduced the concepts of Generalized nets and Intuitionistic fuzzy sets.

[37] Luitzen Egbertus Jan Brouwer (1881-1966) was a Dutch mathematician. He was the founder of the mathematical philosophy of intuitionism.

intuitionistic logic. This criticism was formulated by several authors [29–31], most of them participants of the "Linz Seminars". Unfortunately, in his reply to [29], Atanassov [32] insisted to keep this misleading name. Moreover, the set of truth values of "intuitionistic" fuzzy sets is isomorphic to those used for several other generalizations of Zadeh's fuzzy sets, so we witness the sad fact that well-known results are reproved under "new" and sometimes fancy names. Obviously, this does not contribute to a better standing of the "fuzzy" community among, e.g., mathematicians. But to complete the picture, it should be mentioned that Brouwer's intuitionistic logic was properly carried over to the fuzzy case by G. Takeuti [38]and S. Titani[39] [33,34] – these are truly "intuitionistic fuzzy sets".



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L-Fuzzy Sets and Isomorphic Lattices: Are All the "New" Results Really New? †

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Received: 23 June 2018; Accepted: 20 August 2018; Published: 23 August 2018

Abstract: We review several generalizations of the concept of fuzzy sets with two- or three-dimensional lattices of truth values and study their relationship. It turns out that, in the two-dimensional case, several of the lattices of truth values considered here are pairwise isomorphic, and so are the corresponding families of fuzzy sets. Therefore, each result for one of these types of fuzzy sets can be directly rewritten for each (isomorphic) type of fuzzy set. Finally we also discuss some questionable notations, in particular, those of "intuitionistic" and "Pythagorean" fuzzy sets.

Figure 9: A new article of Klement and Mesiar in 2018.

Rudolf Seising: You mentioned Elbert Walker and his wife Carol. Elbert passed away earlier this year. In 2015, the two gave me an interview for this journal.[40] I asked them: "There are many examples of

[38] Gaisi Takeuti (1926-2017) was a Japanese mathematician. At the *Institute for Advanced Study* (IAS) in Princeton he studied under Kurt Gödel. He is well known for his work in proof theory.

[39] Satoko Titani is with the Chubu University (Kasugai, Japan).

[40] Rudolf Seising: From Fuzzy Sets to Type-2 Fuzzy Sets – an Interview with Carol and Elbert Walker, *Archives for the Philosophy and History of Soft Computing*, vol. 1 (2015), pp. 1-11. URL: http://aphsc.org/index.php/aphsc/article/view/28/21 (16.08.2018)

mathematicians and particularly statisticians who argued that Fuzzy Set Theory is a useless theory and that probability theory and statistics cover all sets of problems that Fuzzy Set Theory solves in a different way. How do you remember these discussions and what do you think about it today?" What is your answer to this question?

Peter Klement: I always had a bad feeling with this type of controversies. The claim that one method or field is the only one when it comes to solve a real problem is usually hard or even impossible to prove in a strict mathematical sense. And how to approach the solution of a given problem may heavily depend on the context in which the problem occurs, on the philosophical background and on the education of the problem solver, and sometimes it may simply be a matter of taste.

A major contribution of Lotfi Zadeh was that – compared with his predecessors – he combined his step towards many-valuedness with a clear view of the practical aspects of his ideas (even in his first paper [2] he mentioned, among others, applications in pattern recognition). It took some thirty years until Petr Hájek formulated his concept of BL-logics [21], a complete and sound system of many-valued logics based on left-continuous triangular norms (i.e., with a residual implication), thus combining Zadeh's ideas with some of tools proposed by Karl Menger [4] when he tried to extend the classical triangle inequality to the case of statistical (probabilistic) metrics.

And earlier this year, Patrik Eklund,[41] Javier Gutiérrez García,[42] Ulrich Höhle, and Jari Kortelainen[43] presented their book [35] where they formulated a mathematical (or, more precisely, an algebraic) language for the phenomenon of many-valuedness, without referring to many-valued logics. These are two impressive pieces of mathematical work whose impact to the modeling of uncertainty has not yet been fully recognized.

On the more applied side, let me mention that fuzzy-based rule-based systems (as in control or pattern recognition where – incomplete or even partially contradictory – expert knowledge is used) in their simplest form are interpolating between points of knowledge, but they have two important properties: first of all, they can be combined with virtually all other methods (systems of equations and inequalities, artificial neural networks, genetic

[41] Patrik Eklund is Professor in Computer Science, Umeå University, Sweden

[42] Javier Gutiérrez García is with the Departament of Mathematics at the University of the Basque Country.

[43] Jari Kortelainen is Principal lecturer at the South-Eastern Finland University of Applied Sciences.

algorithms and genetic programming, machine learning, deep learning, ...) and, secondly they have a high degree of interpretability (as opposed to the black box character of, e.g., artificial neural networks).

Rudolf Seising: So what is your view on the development of all the disciplines in Computational Intelligence and machine learning, which you just mentioned here? Is this a "new Artificial Intelligence" and will this change the character of our concept of Science?

Peter Klement: I am not a specialist in Artificial Intelligence and its development, and I have seen already several waves and fashions claiming to be the ultimate steps of it (in the 1990's fuzzy logic was one of them). Some of the results of deep learning are quite impressive, but I continue to be convinced that clever "hybrid" combinations of analytical models, soft computing and machine learning may still lead to better results. Thinking in terms of applications, I would say that we will have autonomous cars and robots in the near future. But even when the technical problems are solved: legal aspects (who is responsible for an accident between two autonomous cars?) and the "psychology" of robots and of man-machine interaction also need to be solved. Such concerns are not really new: recall that the Czech author Karel Čapek[44] in his drama "R.U.R." (1920) not only coined the term "robot". R.U.R. ("Rossum's Universal Robots"[45]) is a factory which produces "robots" as cheap workers without any rights, and their increasing use results in a drastic change of the world economy. But then the robots rebel against mankind and finally destroy the human race ...

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[44] Karel Čapek (1890-1938) was a Czech writer, photographer, and art critic.
[45] The play R.U.R. appeared in 1920.

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