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Fuzziness in Italy – Traces of a scattered history

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Abstract The history of Fuzziness in Italy is varied and scattered among a number of research groups. As a matter of fact, "fuzziness" spread in Italy through a sort of spontaneous diffusion, and, also subsequently, no one felt the need to create some "national" common structure like an Association or similar things. Since a cohesive retelling would be next to impossible, a few members of the Italian fuzzy community have been asked to recount their experience and express their hopes for the future.

1 Introduction

The history of Fuzziness in Italy is varied and scattered among a number of research groups. As a matter of fact, "fuzziness" spread in Italy through a sort of spontaneous diffusion, and, also subsequently, no one felt the need to found some "national" common structure like an Association or similar things. The European Association - at its level - played, later, a brilliant role. Since a cohesive retelling would be next to impossible, a few members of the Italian fuzzy community have been asked to recount their experience and express their hopes for the future. Although partial and, surely, incomplete, what follows can provide an idea of the richness and variety of the paths followed. A richness due perhaps to just the chaotic and unplanned way of its diffusion.

Since not even a chronological order (let alone a semantic or prioritising one) can be attempted, the contributions are grouped following loose thematic connections mixed with author's lexicographic order.

2 Gianpiero Cattaneo: Brower–Zadeh structures according to a personal approach

This small note will be an information document about my interest developed in several years of research about fuzziness, with induced results on roughness. All started around the seventies of the last century that I spent as an associate professor at the University of Calabria (south Italy), where a colleague from the Department of Sociology introduced me to the world of fuzziness by the knowledge of two, for me very important, papers about this argument: the seminal Zadeh's paper [1] and a De Luca–Termini contribution [2]. Before this, I was involved in theoretical researches about foundations of quantum mechanics according to the so-called unsharp approach, especially from the quantum logical point of view, but I immediately realized a deep formal analogy between the mathematical structures involved in these two approaches where the classical dichotomy crisp–fuzzy corresponds to the quantum dichotomy sharp–unsharp. It is only a terminological difference in describing the relationship between certain and uncertain, precise and unprecise, sharp and vague in two different contexts.

Formally, I was interested to the abstract approach to these arguments based on the algebraic structure of poset, which is a lattice in the fuzzy set theory and a partial ordered structure which is not a lattice in the quantum case. But, in studying the above mentioned contributions in fuzzy set theory I was impressed by the fact that in the Zadeh paper [1] the (distributive, complete and bounded) lattice $\Sigma, \wedge, \vee, 0, 1$ is equipped with a Kleene orthocomplementation mapping $' : \Sigma \to \Sigma$, which generalizes the orthocomplementation of standard set theory (for arbitrary $a, b \in \Sigma$, (oc-1) $a = a''$ strong double negation law; (oc-2) $a \leq b$ implies $b' \leq a'$ contraposition law; (K) $a \wedge a' \leq b' \vee b$

Kleene condition). Differently, into the De Luca and Termini paper [2] the lattice Σ can be equipped with a Brouwer (intuitionistic) orthocomplementation mapping $^\sim : \Sigma \to \Sigma$, which is a different generalization of standard orthocomplementation (for arbitrary $a, b \in \Sigma$ (woc-1) $a \leq a^{\sim\sim}$ weak double negation law; (woc-2) $a \leq b$ implies $b^\sim \leq a^\sim$ contraposition law; (woc-3) $a \wedge a^\sim = 0$ non contradiction law).

I was soon intrigued by the investigation of a poset structure $\Sigma, \leq, ', ^\sim, 0, 1$ in which both these orthocomplementations are involved, with a further interconnection rule about these two, formalized as the law: (ir) for every $a \in \Sigma$, $a^{\sim\prime} = a^{\sim\sim}$, which is satisfied both in standard fuzzy set theory and in unsharp quantum mechanics. The poset requirement, instead of the lattice one, is due to the fact that in the quantum case the partial order does not induce the lattice operations. Notice that in the lattice case, where the lattice meet $\wedge$ and join $\vee$ are algebraic realizations of the logical connectives And and Or, the orthocomplementations $'$ and $^\sim$ are also interpreted as two kinds of non standard negations in the algebraic semantic of some propositional logic, in [3] interpreted as Not and Falsehood (sometimes this latter also as Impossibility).

The first approach to this kind of structure in discussing arguments related to fuzziness can be found in [4], but during my staying in Calabria I started a collaboration with two researchers/collaborators of this university which produced several papers about this poset (resp., lattice) structure, from me defined as Brouwer-Zadeh (BZ) poset (resp., lattice). The first in time was the paper [5], presented in the first Napoli meeting on Mathematics of Fuzzy Systems (1984), where taking inspiration from the concrete case of the lattice–algebraic structure of fuzzy sets on a reference space ($\Sigma = [0,1]^X$, with X the reference space), the general BZ poset structure is introduced, with the analysis of all the formal induced machinery: the definition of crisp (exact, sharp) elements as those elements that coincide with their double Brouwer negation ($f \in \Sigma$ is crisp iff $f = f^{\sim\sim}$), where the two nonstandard orthocomplementation coalesce in a unique standard one ($\forall f$ crisp $f' = f^\sim$), satisfying besides the strong double negation and the contraposition laws, also the non contradiction and the excluded middle laws.

On the basis of this structure, the two modal operators obtained by suitable compositions of orthocomplementations, necessity $\nu(a) = a'^\sim$ and possibility $\mu(a) = a^{\sim\prime}$, which are both crisp elements, are introduced showing that they satisfy the semantical algebraic properties required by modality, but in the context of a non Boolean lattice, such that both coalesce to the identity on crisp elements ($\forall f$ crisp $\nu(f) = \mu(f) = f$). Moreover, the collection of all necessity–impossibility pairs $(\nu(a), \mu(a)') = (\nu(a), \nu(a'))$ gives rise to a three–valued Lukasiewicz poset, with corresponding BZ structure since also in this case it is possible to introduce a Kleene negation $-(\nu(a), \nu(a')) = (\nu(a'), \nu(a))$ and a Brouwer negation $\sim (\nu(a), \nu(a')) = (\nu(a'), \mu(a))$ (as consequence of the

modal required relation between possibility and necessity $\mu(a) = \nu(a')'$). If the original BZ structure Σ is a lattice, then also the induced three–valued Lukasiewicz structure is a lattice.

In the same meeting also the first contribution to concrete BZ poset structure in the unsharp quantum mechanical situation has been presented in [6], where the role of fuzzy elements are played by the so-called effect operators on a complex separable Hilbert space, which as said before has a structure of BZ poset which is not a lattice.

All the BZ definitions, in the poset and not lattice context, are introduced with the investigation of the corresponding properties. In particular, it is shown that the collection of sharp elements coincide with the set of all orthogonal projections, which is an orthomodular lattice despite the original structure is a poset. This leads to the fact that also the induced three-valued Lukasiewicz BZ structure is a lattice.

As widely discussed in some papers of physical content [7,8], the BZ poset of effect operators can be related to the Ludwig approach to unsharp quantum mechanics, whereas the induced three-valued Lukasiewicz BZ lattice is the natural environment of the Piron approach to the same argument.

After an abstract study about non usual orthocomplementations in the fuzziness context [9], the main paper about a treatment of BZ poset structures has been done in [10], and for the specific quantum case see also [11].

In 1991 I started a collaboration with the group of Logics from Florence, directed by Marisa Dalla Chiara and with some of her collaborators, Roberto Giuntini and Federico Laudisa, about BZ structures applied to the logics of unsharp quantum mechanics. I mention here the first contributions to this argument [12–14], with interesting application to physics [15], and a survey which can be found in [16]. In the same period I moved to the University of Milano (from 1984 to 1999) and then to the new University of Milano-Bicocca (from 1999 till my retirement occurred some years ago). In these two new contexts I created new research groups whose main interest was the connection of fuzzy set theory with rough set theory, always in the algebraic context of BZ structures.

As discussed above, a BZ structure, either of poset (unsharp quantum case) or of lattice (fuzzy set case), with the two non standard Kleene a' and Brouwer $a^\sim$ orthocomplementations can be considered the right abstract environment to describe fuzziness. The induced two operators of $\nu(a) = a'^\sim$ and $\mu(a) = a^{\sim\prime}$ are not only the algebraic realizations of modal necessity and possibility, but have the interesting property of being the best lower and upper crisp approximations, respectively, of the approximable element a. This means that on the pair (Σ, Σ_e) consisting of the BZ lattice Σ, now interpreted as the collection of the elements which can be approximated, and the ortholattice Σ_e of corresponding crisp elements, considered as the exact ones, for any approximable element $a \in \Sigma$ the pair $r(a) = (\nu(a), \mu(a))$ is its rough approximation by exact

elements since $\nu(a)$ (resp., $\mu(a)$) is the best approximation from the bottom (resp., top) by crisp elements. Indeed, $\nu(a) \in \Sigma_e$ (resp., $\mu(a) \in \Sigma_e$) is a crisp element that furnishes an inner (resp., outer) approximation of a since $\nu(a) \leq a$ (resp., $a \leq \mu(a)$) and which is the best one since if $f \in \Sigma_e$ (resp., $g \in \Sigma_e$) is another inner (resp., outer) crisp approximation of a, i.e., $f \leq a$ (resp., $a \leq g$), then $f \leq \nu(a)$ (resp., $\mu(a) \leq g$).

A first exposition of these new version of BZ structures can be found in [17], whereas the study of a generalization of this argument can be found in [18] (and see also [19]). Always linked to this argument of roughness in the abstract context I mention the following papers [20–22], with an interesting contribution about the discussion of the role of intuitionistic negation in the Atanassov approach to the so-called intuitionistic fuzzy sets [23]. I conclude with the last citation [24], in which instead of considering as original structure the one based on a Kleene and Brouwer orthocomplementations, the starting point is a structure, called of MDS5 algebra, based on a de Morgan orthocomplementation (i.e., negation) and a modal necessity operator satisfying axioms N, K, T, and 5 of modal logic (which does not coalesce to the identity since the negation is not the Boolean one). The consistency of this algebraic model is proved, since the usual fuzzy set theory can be equipped in a natural way with a MDS5 structure.

3 Antonio Di Nola: Fuzzy Logic as a Logic: the wisdom of fuzzy sets

The novelty of Zadeh' idea of *Fuzzy Set* lies not just in providing a theory for vague concepts, but in offering a representation of them via functions. Fuzzy logic was conceived by L. Zadeh in 1965, [1], as an attempt at dealing with vagueness, with a marked slant towards practical applications. Subsequent developments in that field were mostly driven by engineering practice, intuition, and persistence.

Just to remark the importance of the theory of fuzzy sets, we note that every time that one cannot give a sharp input to a system then a more general framework is needed for the system. This, sometime, can be done using fuzzy systems. Such a machinery helps to cover more cases than classical systems do. Thus the problem of a theoretical foundation of fuzzy sets and of fuzzy systems arises.

To approach any theoretical problem it is important to have open mind to use several different theoretical tools.

It is important to remember that in science "tout se tient". That is, same ideas can appear under many different dresses and many components play a proper role in the picture.

Representation calls for a formal treatment. Formalization is abstraction,

and abstraction, as output, gives several incarnations of the same concept. For example, look at Boolean Algebras. Boolean algebras are the algebraic models of Classical Logic and their role, in mathematics, is relevant in Ring Theory, Topology, Functional Analysis and Probability, among other fields of Mathematics. This means that the deep structure of classical logic is reflected into deep mathematical objects in many different ways. Any generalization of classical logic has to pay a price to the profound interconnection between its algebraic treatment, by comparing the structure of its Lindenbaum Algebra and other mathematical structures. The rich universe of symmetries of Boolean Algebras, in some extent is lost, but, in contrast, new links between Lindenbaum algebras of non classical logics and other field of Mathematics appear.

The Min-Max fuzzy logic, as a first proposed fuzzy logic, had a too restrictive point of view, as it is testified by the very simple lattice structure of fuzzy sets as functional model of such a logic.

A great effort was done to provide algebraic semantics for fuzzy logic, in view to help people, oriented at applications, to better understand the fuzzy logic, as a logic, and to capture an elegant way to mathematically express the wisdom of the idea of fuzzy set.

One may ask: Why fuzzy sets? A first answer could be: to represent a special kind of information. How to use fuzzy sets? By using the mathematics of fixed classes of fuzzy sets. That is to represent the semantics of a fuzzy logic by an algebra of functions. Thus, fuzzy sets codify the information expressed by a precise language, starting from fixed axioms of a logic. The set of axioms represents the basic knowledge.

Many-valued logics are the logical instrument to use when dealing with more than two truth values. In particular, in the large class of many-valued logics, developed so far, a leading role has played by Lukasiewicz Logic. This logic is based on the Lukasiewicz t-norm as interpretation of conjunction.

Besides finding applications to the treatment of uncertain information, the infinite-valued calculi based on Lukasiewicz logic are very interesting for their deep relationships with diverse mathematical areas.

Here, we aim to show that Lukasiewicz Logic and its extensions are generalizations of classical logic which represent a suitable compromise between generality and profoundness of their algebraic models as MV-algebras, for example.

At early stage of fuzzy sets' age soon some interesting ideas arose. We like to mention the idea due to De Luca and Termini ([25]) of *entropy of a fuzzy set*, idea which, later on, was developed by Mundici, [26], in a theory of states of algebras of Lukasiewicz Logic, as formalization of the concept of averaging truth values. Belluce in the paper [27] linked fuzzy sets with Lukasiewicz Logic, and in the paper [28] were proposed non-standard fuzzy sets, showing that by representing vagueness via functions one can consider also infinitesimals as truth values of a vague proposition. Some years later, Hajek ([29]) giving the basic

theory of Fuzzy Logic as a logic, via t-norms operators, included Lukasiewicz Logic among the basic fuzzy logics.

All the above theories, starting from the end of the eighties, were developed also by people at Naples University, where a numerous research group was very active. It is worth to mention, among others, the work of A. Lettieri, S. Sessa, and A. Ventre. A little bit later, and still in Naples, the important seminar: *Order in Algebra and Logic* was held along ten years. The seminar had, among its interests also problems concerning the foundation of theory of fuzzy sets. The seminar was supported by the generosity of the Istituto Italiano per gli Studi Filosofici and by the work of the steering committee: E. Casari, M. Curzio, A. Di Nola, A. Macintyre, D. Mundici, G. Tallini. Several editions were guested abroad, at Oxford University, University of Galway, Barcelona, Marseille and of course at Naples. The neapolitan research group in fuzzy logic was essentially involved in studying Lukasiewicz Logic and MV-algebras.

MV-algebras, a non-idempotent generalization of Boolean algebras, have an impressive array of deep connections with several fields of mathematics, including inter alia lattice-ordered groups, approximately finite-dimensional C*-algebras, and rational polyhedral geometry. The interplay between algebra, geometry and proof-theory in Lukasiewicz logic, and its applicability to the management of information that is both vague and uncertain, are characteristic features of this kind of logic.

Establishing connections between MV-algebras and other areas of mathematics was a mainstream of investigation, see [30], [31].

In [32], Mundici found a categorical equivalence, Γ, between MV-algebras and lattice ordered abelian groups with strong unit. This theorem is the origin of a considerable number of remarkable results in the theory of MV-algebras. In the same vein, a different categorical equivalence was established in [33], where it was shown that the category of lattice-ordered abelian groups is equivalent to the category of perfect MV-algebras.

The representation theory of MV-algebras has a rich history. C.C.Chang proved the first representation theorem, [34], [35] . In the context of many-valued logic, the algebras of [0, 1]-valued functions have a special role. These are the semisimple MV-algebras, i.e. algebras without infinitesimals. Their functional representation was pointed out by Chang and Belluce. In [36] was provided a functional representation for all MV-algebras: any MV-algebra can be seen as an MV-algebra of functions from a set to an ultrapower of [0,1].

3.1 Scaling propositions: beyond groups

At the beginning of the new millennium the neapolitan research group on fuzzy logic was embodied in a larger gruop at the University of Salerno. There, new and younger people like B. Gerla, G. Lenzi, C. Russo and L. Spada started to

cooperate still on studying MV-algebras theory and their links with geometry, topology and functional analysis.

A further research direction was to characterize the class of structures generated by $[0, 1]$ in the language of MV-algebras enriched with the real product. The analogue Mundici's theorem for these structures, called PMV-algebras, was obtained in [37].

It seems quite natural to introduce *modules* over PMV-algebras. The structure of MV-modules is defined and studied in [38]. The MV-modules over $[0, 1]$ are particularly interesting, since they are categorically equivalent with a class of Riesz spaces.

The deep connection between Riesz MV-algebras and Riesz spaces with strong unit allows one to obtain new logical information using geometric and analytic methods. An analysis of the different aspects of this logic, from one side, provides the limit of an intermediate logic, the Rational Lukasiewicz Logic, on the other side MV-Riesz Logic, see [39], can be used to develop a logic for compact Hausdorff spaces via their categorical equivalence with norm-complete Riesz MV-algebras. All these examples witness that Riesz MV-algebras are a malleable notion, in need of a systematic investigation which could lead to significant advancements in the framework of Lukasiewicz Logic.

3.2 The geometry of fuzzy logic

Several studies are devoted to the concepts of algebraic geometry over fields applied to the theory of MV-algebras. According to Mundici, [40], rational polyhedra are the genuine algebraic varieties of the formulas of Lukasiewicz Logic, in a precise sense: zerosets of McNaughton functions coincide with rational polyhedra, see [40]. On the other hand, there are reasons to be interested in other MV algebras, because every MV algebra can be viewed as the Lindenbaum algebra of some many-valued logic, and as such, it has logical relevance. This is why some people from the research group on Logic at University of Salerno generalized somewhat the theory of Mundici to MV-algebras as general as possible, using the main tools of universal algebraic geometry, as developed in [41]. In [42] are shown the first results in this very promising direction.

4 Giangiacomo Gerla: my interest toward fuzzy logic

From the Eighties of last Century I have addressed my interest toward fuzzy logic as a consequence of contacts with the research group composed by A. Di Nola, A. Lettieri, S. Sessa and others at the Federico II University in Naples. This interest was originated by a more general interest in the naïve science of the common man which is qualitative in nature, in contrast with modern science, quantitative in nature. Indeed, at the same time, I investigated the possi-

bility of defining mathematical models of the naïve idea of space in which one assumes as a primitive the notion of region or solid body while the points are defined by suitable sequences of nested regions (*point-free geometry*).[1] These two interests are related since, as in some recent papers I emphasizes, fuzzy logic appears to be a promising tool to face point-free geometry and this provided one refers to languages containing predicate symbols such as "small", "close", "contained" whose intended meaning is geometrical in nature.

More specifically, I embrace the graded syntax approach to fuzzy logic since, in my opinion, it fits well with the argumentations in our everyday lives. This means that I refer to the famous paper by J. A. Goguen [44] and with the successive formalization proposed by J. Pavelka (see also the significant contributions of V. Novak [45]). In accordance with Pavelka, Gerla's researches are based on the systematic use of the theory of closure operators in a complete lattice (see [46]). This means that if F denotes the set of formulas of a first order language, then a model is identified with a fuzzy subset m of F, i.e. the fuzzy subset of formulas which are true in this model. In accordance, a semantics is a class $M \subseteq [0,1]^F$ of fuzzy subsets of F. In several cases, M is the class of truth-functional valuations of the formulas in F. Now, in the graded approach instead of a set of axioms one considers a fuzzy subset $\tau : F \to [0,1]$ of axioms. The intended meaning is that, for every formula α, $\tau(\alpha)$ is the constraint $[\tau(\alpha), 1]$ on the truth value of α, i.e. the information "the truth value of α is greater or equal to $\tau(\alpha)$". In accordance, one says that $m \in M$ is a *model of* τ provided that $m \supseteq \tau$ and the *logical consequence operator* $C : [0,1]^F \to [0,1]^F$ is defined by the equation

$$C(\tau)(\alpha) = inf\{m(\alpha) : m \in M, m \supseteq \tau\}.$$

$C(\tau)(\alpha)$ represents the best possible constraint on the truth value of α we can derive from τ. Notice that in this formalism one refers to the fuzzy subset of logically true formulas $C(\emptyset)$ and not to a classical subset of logically true formulas as in the traditional syntax approach to fuzzy logic.

A second operator is the *deduction operator* $D : [0,1]^F \to [0,1]^F$. Namely, the notion of a proof π of a formula α under the fuzzy subset τ of hypotheses is defined and a constraint $c(\tau,\pi) \in [0,1]$ is associated with π, given τ. Since different proofs of α can give different constraints, $D(\tau)(\alpha)$ is obtained by fusing all the possible constraints,

$D(\tau)(\alpha) = sup\{c(\pi) : \pi$ is a proof of α under the fuzzy subset τ of hypotheses$\}$.

Both C and D are closure operators.

[1] Both naïve logic and naïve geometry find their place alongside 'naïve physics' (see [43]).

Now, given a fuzzy semantics, the basic question is to find a deduction apparatus whose associated deduction operator D coincides with C (completeness theorem). It is evident that this operator should be in some way 'computable'. As matter of fact, the meaning of the completeness theorem for a logic is that the set of theorems of a decidable theory is effectively enumerable. In accordance, I propose to extend to the fuzzy framework the classical notions of 'computable', 'decidable', 'effectively enumerable', 'enumeration operator'.

At this regard, my idea is that to do this, we have to take into account the strong difference between classical logic, based on the finite set $\{0, 1\}$, and fuzzy logic, based on to the continuum [0,1]. This difference entails that while classical logic is related with algorithms giving the output after a finite number of steps (recursive arithmetic), fuzzy logic must be related with endless effective processes of approximation (recursive analysis). Then, he proposes the following definitions (see for example [47]).[2]

Denote by $[0,1]_Q$ the set of rational numbers in [0,1] and let S be a nonempty set. Then a fuzzy subset $s : S \to [0,1]$ of S is *effectively enumerable* (*co-enumerable*) if there is a recursive map $h : S \times N \to [0,1]_Q$ such that

i) h is order-preserving (order-reversing) with respect to the second variable,

ii) $s(x) = lim_{n \to \infty} h(x,n)$ for every $x \in S$.

A fuzzy subset s is *decidable* provided that is at the same time effectively enumerable and effectively co-enumerable. Notice that in this case, $s(x)$ can be any computable real number. Now, the kind of effectiveness required to the deduction operator D of a fuzzy logic requires that if τ is decidable, $D(\tau)$ has to be effectively enumerable. This leads to the following definition. Denote by $FFS(S)$ the class of finite fuzzy subsets of S with rational values, put $SEQ_f = FFS(S) \times F$ and denote by $s_f \vdash \alpha$ an element of SEQ_f. Moreover, define the relation $\ll$ by assuming that $s \ll s'$ provided that $s(x) < s'(x)$ for every x such that $s(x) \neq 0$. Then, $H : [0, 1]^S \to [0, 1]^S$ is an *enumeration fuzzy operator in S* provided that there is an effectively enumerable fuzzy subset $w : SEQ_f \to [0,1]$ of SEQ_f with rational values such that

$$H(s)(x) = Sup\{w(s_f \vdash x) : s_f \ll s\}.$$

I observe that the deduction operators in the main fuzzy logics is an enumeration operator. This entails that if τ is decidable, then $D(\tau)$ is effectively enumerable as required. In particular $D(\varnothing)$ is a recursively enumerable fuzzy subset and, in my opinion, this shows that the graded approach to fuzzy logic is immune of the strong difficulties arising from Scarpellini's theorem [48]. Indeed, notice that the fact that $D(\varnothing)$ is effectively enumerable is compatible

[2] My researches on the closure operators and on fuzzy computability have been largely developed in collaboration with L. Biacino of the Federico II University. More recently, several papers where written in collaboration with Cristina Coppola and Tiziana Pacelli of Salerno University.

with the fact that $\alpha \in F : D(\emptyset) = 1\}$ is not.

5 Giulianella Coletti: A bridge between fuzzy set theory and coherent conditional probability: a step toward armonizing theories for managing uncertainty

I'm a member of a group of researchers in probability theory, which follows the ideas, the suggestions and intuitions of de Finetti. As is well-known de Finetti's approach to probability (and in particular to conditional probability) trough coherence is particularly suitable for managing uncertainty in intelligent systems and in approximate reasoning. During the nineties we actively attended various international conferences in soft computing, where the different approaches to reasoning under partial, imprecise and vague information were animatedly discussed.

Previously I had come in contact with fuzzy set theory during interesting discussions with Settimo Termini, which at the end of the eighties was in Perugia as Professor of Cybernetics and Information Theory. However, fuzzy sets and fuzzy logic were still to me essentially an intellectual curiosity, an enrichment of my knowledge, but certainly not a research field. Instead, during the meetings mentioned before, Romano Scozzafava and I realised that it was necessary to have a single well-founded setting for handling problems such as the following:

> *A box contains n balls of various sizes $s_1, ..., s_m$, with $m \leq n$. Under the hypothesis that "most balls are large", what is the probability that a ball drawn at random is small*?

The possible strategies for this aim were two:

- Define conditional probability in a not Boolean ambit, following the line of the logic school (see for instance Mundici [32,49], Hajek, Tulipani [50]) and find which results of probability theory continue to hold in this context, that in any case was limited to Lukasiewicz t-norm and t-conorm, surely not the best for managing probability problems.

- Propose an interpretation of fuzzy sets and fuzzy events in terms of conditional events and conditional probability, and use all the results in probability theory to manage probability on fuzzy events.

The second idea seemed to be the most profitable and so for some years Romano Scozzafava and I, with the important contribute of some of our pupils (in particular Barbara Vantaggi) coped with the construction of a bridge between the two theories. For that it was necessary to develope a model of

generalised inference, based on the theory of coherent conditional probabilities (essentially introduced by de Finetti) and the relevant extension problems (see [51–60]). Such theory is based on the following milestones:

– A conditional event is any ordered pair of Boolean events $E|H$, with $H \neq \emptyset$, which can be both true or false and are different only in the role they play (H acts as an hypothesis). It is in fact essential to regard also the conditioning event H as a *variable*, i.e., the status of H in $E|H$ is not just that of something representing a given fact, but that of an uncertain event (like E) for which the knowledge of its truth value is not required.

– The uncertainty can be caused by different circumstances, but it is reducible in fact to a lack of complete information.

– The theory of reference (coherent conditional probability) is able to handle both domains without particular Boolean structure and partial or imprecise assessments.

In this context we proposed our interpretation of a fuzzy subset: we refer to the state of information (at a given moment) of a real (or fictitious) person that will be denoted by "You". Let Z be a (not necessarily numerical) variable, with range C_Z, denote by z the event $\{Z = z\}$, by φ any *property* related to Z and finally by E_φ the Boolean event {You claim that Z is φ}. Consider the conditional events $E_\varphi|z$, $z \in C_Z$ and a coherent probability assessment on them. By a fundamental result in coherent probability theory, it follows that You may assign to each of these conditional events a degree of belief $P(E_\varphi|z)$, without any syntactical restriction.

So, we can define a fuzzy subset E^*_φ of C_Z as a pair $\{E_\varphi\ ,\ \mu_{E_\varphi}\}$ with

$$\mu_{E_\varphi}(\cdot) = P(E_\varphi|\cdot).$$

For this interpretation the semantic of the membership degree of z is very clear: $\mu_{E_\varphi}(z)$ is the probability that You claim that Z is φ, in the case that Z assumes the value z. Moreover, by using only results of coherent conditional probability theory, we could find many (unfortunately not all) concepts and the related results of the theory of fuzzy sets, such as the following: given two fuzzy subsets $E^*_\varphi,\ E^*_\psi$ related to the same variable Z, with the events $E_\varphi,\ E_\psi$ logically independent, the coherent values for $\mu_{\varphi\wedge\psi}(z) = P(E_\varphi \wedge E_\psi|z)$ and $\mu_{\varphi\vee\psi}(z) = P(E_\varphi \vee E_\psi|z)$, for any given z in the range of Z, can be obtained by any Frank t-norm and their dual t-conorms.

Just for citing one of the mentioned limitations of this interpretation, I recall that: coherence is violated if any t-norm and t-conorm different from those in the Frank's class are used for computing $\mu_{\varphi\wedge\psi}(z)$ and $\mu_{\varphi\vee\psi}(z)$.

Nevertheless the most important goal we obtained is that this proposal makes possible, without introducing new concepts, but only using probabilistic tools, to solve problems containing randomness and fuzziness [61–64] such as the following (proposed to me by Zadeh) referable to a Bayesian updating of an initial probability [65]:

- *A and B are boxes, each containing 20 balls of various sizes. Most of the balls in A are large, a few are medium and a few are small. Most of the balls in B are small, a few are medium and a few are large. The balls in A and B are put into a box, C. What is the probability of drawing a ball from C which is neither large nor small*?

- *Usually it takes Robert about an hour to get home from work. Usually Robert leaves office at about 5pm. What is the probability that Robert is home at 6:15pm?*

Our research in this ambit is not concluded: recently we proposed the same scheme, by using different measures of uncertainty, precisely coherent conditional plausibility [66] and coherent conditional possibility [67]. The interest for these studies comes from the necessity to manage real situations, where the information about the uncertainty of the elements $z \in C_Z$ is expressed by a non additive uncertainty measure. Nevertheless the results we obtained are not so close to the classical theory of fuzzy sets, such as those obtained by coherent conditional probability.

6 Mario Fedrizzi: Consensus modeling in group decision making

Most of the work carried on by the research team of the University of Trento leaded by Mario Fedrizzi (Silvia Bortot, Matteo Brunelli, Michele Fedrizzi, Ricardo Alberto Marques Pereira) was focused on the modeling of consensual processes in group decision making. It's well known that the notion of consensus plays a key role in modeling group decisions, and for a long time it was meant as a strict and unanimous agreement, however, since various decision makers have different more or less conflicting opinions the traditional strict meaning of consensus is unrealistic. The human perception of consensus is much "softer", and people are willing to accept that a consensus has been reached when most or the more predominant actors agree on the preferences associated to the most relevant alternatives. The "soft" meaning of consensus, advocated as realistic and humanly consistent, can lead to solve in a more constructive way group decision making situations by using modeling tools based on fuzzy logic. Accordingly,the problem of consensus reaching modeling in a fuzzy environment was addressed for first in [68,69], as based on individual reciprocal fuzzy preferences, [70–72]. The basic framework within which these consensus models have been developed can be depicted in the following way: There is a set of decision makers or experts who present their opinions concerning a set of alternatives and these alternatives may initially differ to a large extent. If the individuals are rationally committed to consensus, via some exchange of information, bargaining, etc. the individuals' opinions can be modified and the group may get closer to consensus. Here consensus is meant as the degree to which most of the individuals agree as

to their preferences concerning almost all of the relevant opinions. This degree of consensus takes on it values in the unit interval, and it's more realistic and human consistent than conventional degrees, mostly developed in the probabilistic framework. This approach was then extended in a dynamical setting in [73–75]. In this new model, the consensus reaching dynamics is generated starting from a combination of a soft measure of collective disagreement with an inertial mechanism of opinion changing aversion. It acts on the network of single preference structures by a combination of a collective process of (nonlinear) diffusion and an individual mechanism of (nonlinear) inertia. The overall effect of the dynamics is to outline and enhance the natural segmentation of the decision makers group into homogeneous preference subgroups. Each decision maker is represented by a pair of connected nodes, a primary node (dynamic) and a secondary node (static). The primary nodes form a fully connected sub network and each of them encodes the individual opinion of a single decision maker. The secondary nodes, on the other hand, encode the individual opinions originally declared by the decision makers and each of them is connected only with the associated primary node. The iterative process of opinion transformation corresponds to the gradient dynamics of a cost function depending on both the present and the original network configurations. The value of the cost function combines a measure of the overall disagreement in the present network configuration and a measure of the overall change from the original network configuration. The various interactions involving each node are mediated by interaction coefficients whose role is to quantify the strength of the interaction. The diffusive interaction between any pair of primary nodes is mediated by the interaction coefficient, whereas the inertial interaction between any primary node and the associated secondary node is mediated by the corresponding interaction coefficient. It turns out that the values of these interaction coefficients are given by the derivative of a scaling function. The diffusive component of the network dynamics results from the consensual interaction between each node and the remaining nodes in the network. The aggregated effect of these interactions can be represented as a single consensual interaction between the given node and a virtual node containing a particular weighted average of the remaining opinion values. The computer simulations of the model, particularly those in the fuzzy preference framework illustrated in [76], provided clear evidence that the fuzzy soft consensus model exhibits interesting non standard opinion changing behaviour in relation to the original crisp version of the model. In [77], in the framework of social networks analysis, we addressed the problem of consensus evaluation by endogenously computing the importance of the decision makers in terms of their influence strength in the network. To this aim, we start from centrality measure and combine it with the fuzzy m-ary adjacency relation approach. In this way, we introduce a flexible consensus measure that takes into account the influence strength of the decision makers according to their

eigenvector centrality. Moreover, we propose an optimization problem which determines the maximum number of the most important decision makers that share a fixed desirable consensus level. In [78] we investigated the relationship between the soft measure of collective dissensus and the multidistance approach to consensus evaluation. The novelty of the contribution consists in the introduction of a particular type of sum-based multidistance used as a measure of dissensus. This multidistance is characterized by the application of a subadditive filtering function whose effect is that of emphasizing small distances and attenuating large ones. An illustrative example is then developed in order to compare the new dissensus measure with the OWA-based multidistance obtained assuming that the weights are linearly decreasing with respect to increasing distance values.

7 Gabriella Pasi: my path in Fuzzy Logic

I read the first papers on Fuzzy Logic in 1990, when I was a researcher at the Italian National Council of Research (CNR). I came across the task of Information Retrieval well before Web search engines' birth, because I joined a research team that was working at defining and implementing an Information Retrieval System (Search Engine) for managing the scientific articles produced by CNR's researchers. In the phase of analysing the task of IR as well as the central role of the concept of relevance of documents to users' information needs, I had the opportunity of reading some papers written by Tadeusz Radecki and by Donald Kraft. In his seminal paper of 1979, titled "Fuzzy set theoretical approach to document retrieval", Radecki proposed a first generalization of the Boolean retrieval model [79]; this proposal was followed later by other authors. The main issue underlying this generalization was that Information Retrieval is a decision making activity which relies on subjective and often vague needs that an automatic system may only partially capture, and not exactly as in the Boolean model. The application of Fuzzy Set Theory (FST) and Fuzzy Logic to IR was initially proposed by several researchers such as Donald H. Kraft, Abraham Bookstein, Ducan Buell, Rita de Caluwe, Etienne Kerre, as an adequate formal framework for modeling the topical relevance of a document to a query as a gradual property, by thus allowing to shape IR systems that can rank documents based on their estimated relevance to user queries [80–82]. Based on this background in 1991 I co-authored (with Gloria Bordogna and Paola Carrara) a paper titled "Query term weights as constraints in fuzzy information retrieval", published in 1991 on the Information Processing and Management Journal [83]. A bit late, in 1992, at the First International Conference on Fuzzy Systems (Fuzz-IEEE) in San Diego we presented a paper titled "Extending Boolean Information Retrieval: a Fuzzy Model based on Linguistic Variables". This event was fundamental in determining a big part of the research activities I did undertake in subsequent years. At that conference I

met people with whom I started beautiful personal and professional relationships. I met Donald H. Kraft with whom Gloria and myself had an interesting working lunch in which we talked about the content of our proposals as well as the links to his works. I met Ronald Yager with whom I had the chance to discuss; as a consequence of the discussions with him, with Gloria we envisioned the usefulness of OWA operators for defining flexible query languages for IR systems, that we first proposed in 1995 in our paper titled "Linguistic Aggregation Operators of selection Criteria in fuzzy information retrieval" [84]. In San Diego I also met Patrick Bosc, Didier Dubois and Henry Prade, Jim Bezdek, Piero Bonissone and several other people who we would have seen and met in numerous subsequent events. Last but not least, I had the chance to meet Lotfi Zadeh, who became an important reference for the evolution of my research experience. What especially surprised me was his kindness, availability and curiosity in talking to me, in giving me advices. Overall I found a greater openness and enthusiasm of this scientific community with respect to other scientific communities. Several years have passed from that conference, and with the impressive development of the World Wide Web the era of big data has emerged, with the related need of defining systems able to provide users with an easy access to information relevant to their specific needs; this "old" research issue has become still more crucial. The uncertain and vague nature of the IR task has been deeply analysed from 1990 to now, and several approaches based on probability theory, machine learning techniques, and natural language analysis have been proposed. In this context, the application of Fuzzy Set Theory to Information Retrieval has been mainly aimed to the definition of retrieval techniques capable of modelling, at least to some extent, the subjectivity, vagueness and imprecision that is intrinsic to the process of locating information relevant to users' needs. Fuzzy Set Theory has proven to be a valuable means to define a variety of approaches, ranging from the definition of flexible query languages, fuzzy associative mechanisms, adaptive document's representations, and personalized systems [85–88]. In particular, Fuzzy Set Theory has been applied in the context of IR to the following main purposes:

- to deal with the imprecision and subjectivity that characterize the document indexing process, i.e., the process by means of which the content of a text is automatically synthesized and represented on the basis of keywords extracted from (or associated with) it;
- to capture and manage vagueness in query formulation: user requests are usually expressed by a few keywords, whose intended semantics can be disambiguated depending on both the user and query context, and whose importance in characterizing the search topics varies depending on several factor among which their specificity, their position within the query string, the collection searched, the user context, and other factors;

- to define "soft" associative mechanisms, such as ontologies, thesauri and algorithms for documents' clustering, which are often employed to extend the functionalities of an IRS.
- to define flexible decision strategies in meta-search engines (for fusing the ranked lists of documents retrieved by several search engines), or in distributed IR, for selecting the most adequate sources of information to a query based on their contents, reputation, user needs;
- to represent and query semi-structured information (XML document repositories) by allowing users to specify degrees of preference on the documents subparts they consider as the most significant to their needs, as well as to specify flexible constraints on both document structure and content.

To summarize, Fuzzy Set Theory has provided early ideas for modelling important features of IR systems, and it can still be promising for shaping search engines of the future. In particular, the notion of aggregation plays an important role in IR; the approaches that make use of the notion of aggregation rely on the interpretation of Information Retrieval (and of the related tasks)as a Multi-Criteria Decision Making (MCDM) problem, from various perspectives. The first and more straightforward perspective is to interpret the overall IR process as a MCDM process aimed at selecting the best alternatives (documents) based on the assessment of the performance of multiple criteria (the keywords specified in a user's query). Another important perspective is related to the assessment of the relevance of a document to a query, which is generally decomposed into the evaluation of the performance of several relevance dimensions (the considered criteria, e.g., topicality, novelty, popularity, recency, etc.). Another process that may require the application of appropriate aggregation operators is the indexing process, when applied to structured documents. Meta-search is another interesting task that can be seen as an instance of a Multi-Expert Decision Making problem, and that may rely on the appropriate choice of an aggregation operator. A quite interesting aspect implied by the previous interpretations of various phases of the IR process is that the choice of different aggregation operators can produce different results. Despite the potential impact of aggregation on the whole IR process, this aspect has not received the proper attention in the literature. Only recently some approaches have appeared demonstrating the importance of this issue, and its potential impact on the search process. Another interesting research issue where Fuzzy Set Theory could play a role is related to memes' identification and tracking. The word "meme" has been coined in 1976 by Richard Dawkins to indicate "the basic unit of cultural transmission or imitation": as well outlined by Susan Blackmore [89]: "Memes, like genes, are replicators, that is, they are information that is copied with variation and selection. Be-

cause only some of the variants survive, memes (and hence human cultures) evolve". As previously outlined, the Web constitutes a huge information repository, where several services and applications represent a rich soil for memes' growth and evolution. Let us think about blogs, e-mails, news resources and social networking services (like Twitter and Facebook). In the context of the Web, the concept of Internet Meme (e-meme) has been introduced as a unit of information (idea) that is replicated and propagated by people using one or more of the previously mentioned applications.

8 Aldo G. S. Ventre: A contribution to Fuzziness in Italy

I became assistant professor at the Institute of Mathematics of the Faculty of Architecture of the University of Napoli (now, University "Federico II") in 1969. In this Institute professors Giulio Andreoli and Angelo Fadini were studying logic of indeterminateness and they called "nebulosi", let say foggy, the induced sets. These researches developed further inspired by the paper "Les ensembles flous en linguistique" by Y. Gentilhomme [90]. The climate was favourable for a development of studies in fuzzy sets. Starting 1975 a strong curiosity led me to exchange ideas with Fadini [91,92] and Aldo De Luca on the topic of fuzzy sets. I was interested in the measure of fuzziness and defuzzification processes; I am grateful to them and Constantin V. Negoita, at that time professor in the University of Bucharest, for suggestions and advices. Antonio Di Nola, from the same Institute, and myself worked in the field of fuzzy entropy and induced structures [93–100] and other colleagues were interested in fuzzy sets: among them let me mention Rosanna Ambrosio, Virginia Cavaccini, Carmela Cella, Anna M. D'Aristotile, Ada Lettieri, Fortuna Liguori, Giulia B. Martini, M. Cristina Miglionico, Rossana Sarno, Salvatore Sessa. Starting the mid-eighties two branches were focused by Massimo Squillante and myself: decomposable measures w.r. to archimedean t-conorms [101], with extensions to classical topics of real analysis, e. g. the range of decomposable measures [100, 102–107] and the consistency for uncertainty measures in order to embed the various aspects of uncertainty in a unique context [108] with applications to financial laws [109] and economics (A.Maturo, Tofan, Ventre). This was a long term project in which also several young people joined. The interaction between the groups of Napoli and Trento, that stems in 1986 and continues until today, is fruitful, especially in the field of synthesis of judgements and multicriteria multiagent decision making, enhancing consensus in committees [110–112]. Further researches on the topic are carried out by Luciano Basile, Bice Cavallo, Livia D'Apuzzo, jointly with, Mario Fedrizzi, Michele Fedrizzi, Ricardo Pereira from Trento. Antonio Maturo, Barbara Ferri, Serena Sanseviero, Fabrizio Maturo, Simonetta Forcini from the University of Chieti-Pescara participate in the activities with Napoli and Trento developing also applications in social sciences and urban planning [113–117]. Relations with

statistics are studied in [118–120] We have fond memories of Pina Varone and Ada Lettieri died prematurely. During the years, relations and cooperation have been established with European and American Universities. Let us just mention the most rooted and lasting joint researches and initiatives with Peter Klement (Linz), Siegfried Weber (Mainz), Patrik Eklund (Åbo Akademi and Umeå) in different theoretical and applicative fields of mathematics and computer science.

9 Settimo Termini: How I began to do (and went on doing) a few fuzzy things

When I was asked to write something about the early years of fuzziness in Italy I thought that this could be an easy enterprise: just try to collect some early memories and put them in the right order, filling up, in some way, when and if necessary, some memory faults, something very probable since I am referring to the end of the Sixties of last Century, when I moved from the University to a CNR Research Laboratory outside University (a novelty for Italy, then) and begin to convert myself from a theoretical physicist to "something else" (it was, then, not clear what was this something else, and still continues to be unclear now[3].

However along the writing I realized that, if taken seriously, the enterprise was not so easy to do. But let's begin. It is easy to say when I heard about fuzzy sets for the first time. It was on 1969. Eduardo Caianiello - back from one of his scientific trips around the world - told me to look for a new interesting idea he had heard from Lotfi Zadeh. He did not have a copy of the paper, had not read it although he told me many details. His knowledge was simply a byproduct of a conversation with Zadeh. Luckily enough he remembered that had appeared on Information and Control "a few year before", so I was able to find it. In that period I had began to discuss scientific questions with Aldo de Luca, another physicist converted to "something else" and so we begin to discuss also Zadeh's paper. We moved along three different paths. The first was the one of "measuring the degree of fuzziness" [25] (the way along which we arrived at this idea is told in [122]), the second path regarded the analysis of the algebraic structures underlying this new idea [2]. But while analyzing

[3]Let me specify that - joking about myself - I refer to a big scientific question. The scientific and epistemological "status" of the so-called "information sciences", of which cybernetics was a prototype. A field that is still evolving and that has produced *wonderful* things in a very *chaotic* way - together with not equally valuable things, as frequently happens amid the chaos. Fuzzy sets are completely immersed in this context inside which they constitute a very meaningful presence. For a few personal remarks see [121]. In my view one cannot put order in this chaos by means of a sort of a *"rappel a l'ordre"* as was done by the artistic milieu at the beginning of XX Century. Information sciences (and, among them, the presence of fuzziness) ask for a radical *Re-Vision* of our traditional "habits of mind".

these purely "technical" aspects we felt that, having encountered a "qualitatively new" notion, a few epistemological reflections were unavoidable [123].

The first two papers appeared, so to say, in the "most adequate" journals. For the third one, we declined Lotfi's invitation to publish it in "Information Sciences". We were convinced that the "right" journal, in this case, was the old Italian *Scientia* founded at the beginning of the XX Century with the specific mission of presenting analyses of new scientific concepts[4].

We were not sure that our paper would be accepted in this venerable journal, and so were very proud when it, finally, appeared. Subsequently, however, we asked ourselves whether, by accepting Lotfi's invitation, the paper could have been read by a wider audience.

Subsequently, the initial ideas - especially those presented in [25] - were technically further extended and developed as, for instance, in [126–128] while the epistemological considerations of [123] were developed in two directions. The first (at the beginning, in a dialogue with Guglielmo Tamburrini) looked for possible paradigmatic connections of the background of the basic ideas of fuzziness with similar methodological approaches present in the development of cybernetics [129] (see also [130] for focusing the context and [121] for subsequent reflections on the context), Something that would be later masterly developed by Rudi Seising [131] (see also [132, 133]). The second tried to face the novelties involved in Lotfi's idea by analyzing them in a comparison with specific different approaches (like, for instance, Alternative Set Theory) [134–136]. Let me add that I looked also for connections with distinct (and distant) notions like "verisimilitude" by discussing with David Miller (these last attempts, I confess, did not produce meaningful results). Along these years, I had contacts and exchanges with a few colleagues and friends (e. g., Gianpiero Cattaneo, Tonino Di Nola, Aldo Ventre) and I also remember a very interesting (and *unusual*) discussion with Marisa Dalla Chiara at a Varenna school on the Foundations of Physics, along which she began to speak with me of the measures of fuzziness - a topic - she told me - which had heard by talking with Aldo de Luca (the paper had not yet been printed). "Unusual" since Marisa, at the beginning of our conversation, had not realized that I was involved in the topic.

After moving to University from the National Research Council, I remember with a great pleasure - during Perugia years (1987-90) - many discussions and scientific exchanges with Silvano Rivoira, Carlo Bernasconi, Giuliana Regoli and, especially, Giulianella Coletti and Romano Scozzafava (see [137,138]), just when the latter two began to pursue the brilliant path which finally produced such important and deep connections between De Finetti approach to probability and Zadeh's proposal (see [55]). Subsequently, from 1990, in Palermo

[4]A journal in which also Einstein and Russell (among many) had written exactly with this aim. (see, http://amshistorica.unibo.it/scientia). Unfortunately, Scientia ceased its publications in 1988 (see [124] and [125]).

when I moved here from Perugia, with the late Vito Di Gesù, an old date friend of mine, who involved me also in one of his periodic meetings [139].

No mention has been done of organizational activities; the contacts and exchanges mentioned above were, in fact, purely personal exchanges. If I can speak of an organizational activity (also this very limited indeed) this was oriented towards foreign contacts especially with Enric Trillas and all the things he begin to organize. Also in this case, I must confess, the driving role was his, not mine. Among these, I want to spend a few words on a Las Palmas Meeting (see [140]). This was, perhaps, the first (and unique) time in which I *did* try to promote a "public" confrontation on the problem of foundations and meaning of fuzzy sets. I thought it was very important to try establishing a link with other attempts at formalizing Vagueness. I worked hardly, from an organizational point of view, for having in Las Palmas the presence of two Czech logician (Jiri Becvar and Antonin Sochor) and asked to Siegfried Gottwald to tell the story of Klaua group. These were difficult things to obtain, before 1989. Unfortunately, however, it was not possible to have, after all these efforts, a true confrontation. The book cited above, published thanks to the efforts of Heinz Skala, fuzzily witnesses this effort.

Let me now explicitly refer to the difficulty I met in writing these pages. The request done to myself and others to present the "entrance" of fuzziness in Italy was not, I thought, concerned only to research and related results "in themselves" but presenting them - also - from an organizational point of view, describing how these new concepts begin to colonize the scientific environment of our Country. The fact that - although for purely causal reasons - Aldo and myself had been the forerunners in this new topic in Italy, would have implied - I thought when I begin writing these *souvenirs* - the duty of promoting and contributing to construct, here, some stable connecting network or national association (as happened elsewere). And *I didn't*. All the contacts listed above - and others, like the one with Silvano Di Zenzo, working at IBM Rome Scientific Center (a good occasion for a confrontation with industrial research, picking up similarities and differences) were not used by myself also in the direction of creating a research network. Was that a lost occasion? And, more importantly, does it point to some not irrelevant questions worth of some deepening? I - now - think that, following Ockham, also Associations "non sunt multiplicanda praeter necessitatem". In fact, the necessity for research is to present (and critically discuss) ideas, *new* ideas, their *theoretical* developments and their *applications* (two things really *strictly* connected, differently from what one usually thinks). And Italian scholars have provided ideas to the new field. The fact that this review is polyphonic, then, not only correctly describes the real situation but reflects what can be considered a right approach. However, let me add a few (personal) comments. One can work for organizational aspects only when it is clear *what* one aims to organize. In those remote years, I was trying to focus the innovative elements of this new field. The fail-

ure of Las Palmas attempt, together with the extreme difficulty of organizing it, is what definitely convinced me that the path towards a clarification could (or, perhaps, should) be pursued on at the level of purely personal contacts. This clarification would have been obtained by me 20 years later [141, 142].

Nothing has been, yet, said about the activities done after my return to Palermo University in 2009, after seven years spent in directing the CNR Institute of Cybernetics, now named to his founder, in which 40 years before I had heard by him the words *fuzzy set* for the first time[5]. Due, perhaps, to the long pause and thanks to the daily scientific interaction with Marco Elio Tabacchi, I was finally able to rethink at the questions posed by fuzziness in a fresh way focusing both on a critical analysis of the formal developments of the last decades and on a few questions posed by the use of this notion in soft sciences, the field that Zadeh had envisaged as the most fruitful for the application of his idea and which, in practice, had been very refractory [144, 145]. More details about these developments will be given by Marco with a fresh perspective.

10 Marco Elio Tabacchi: The transdisciplinary travel of Fuzziness

The first time I came across Fuzzy Sets I was a young and inexperienced student of Computer Science. As an aside of an image processing curriculum my professor spent a couple of hours introducing us to the mysteries of the idea of degree and Fuzzy arithmetic. The whole thing had a sort of secret sect bent, something like the tools that the powers to be don't want you to know and use – a probably unwitting attitude I have seen many times in the community and that has harmed more than helped. On an inexperienced and young student this approach undeniably had a major fascination, further built up by the reading of Kosko's Fuzzy Thinking [146] (of which I still use two chapters as student's material for my AI course, skimmed from the occasional psychobabble). I was fascinated by the elements of oriental philosophy behind the idea, but Vito Di Gesù, the person mentioned above, pointed me on the fact that fuzziness was perfect for describing phenomena that were very complicated to describe by means of traditional approaches and for operating on them without resorting to the intricacies of calculus. He showed me his ideas for a picture search engine based on fuzzy distances – years before google image. This idea resonated with my passion for images, as in a previous professional life I was a fine artist, and from this meeting started my first collaboration to a published scientific paper. The following years of research were dedicated to the appli-

[5] A period in which - due to the duty of following a chain of irrational "reforms" of CNR promoted by the *pro tempore* Government - I had hardly any possibility to do research work in whatever sense one choose to interpret the word. However, this hot climate contributed to force me to reflect on the conditions allowing to do research and guiding the way in which it can be directed and can usefully interact with the economic development of a Country [143].

cation of fuzzy methodologies to classification problems in arts and cognitive science. In the meantime I become the Scientific Director at *Istituto Nazionale di Ricerche Demopolis*, where staples of fuzzy logic such as CWW were applied to the analysis of natural language answers to questionnaires.

The untimely demise of Vito left me without a reference in the fuzzy world, but proving that sometimes lightnings can really strike twice, Settimo Termini of Fuzzy Entropy fame [25] was back in Palermo from his Naples' years (see above) and took me as a collaborator. Settimo's familiarity with the "two worlds" of human knowledge and his sensibility toward the foundational problems of uncertainty have added a new dimension to my research in Fuzzy Sets. We have discussed applications of Fuzzy Set Theory more as a step toward a more coherent theory of uncertainty, which considers not only degrees of ownership and composition thereof, but the inherent imprecision of (any description of) things. And extolled the pervasiveness of such concept not only in the more traditional human sciences, where this is somehow taken for granted, but also in hard science such as engineering and computing. These years spent in Fuzziness research have surely widened my views on uncertainty, and changed my expectations about what precise measurement can really yield – and why is a blessing. Not everything smells of roses, though. The extreme sectorialisation of Fuzziness into a discipline of systems has probably dissuaded practitioners of human sciences, which have often snubbed it, from taking an interest despite there being a willingness for a joint effort. The possible role of Fuzziness as a methodological bridge between the hard sciences and humanities was as such neglected. We have worked long and hard to try and reaffirm this significative aspect of fuzziness [141, 145, 147, 148] still without a lot of luck. Nonetheless, the deep relationship between Fuzziness and the cognitive aspects of human nature survived and resurfaced in a number of methodologies from the more general context of Computational Intelligence (CI). I have married my interests in fuzziness and cognitive science [132, 133], by working on how human performances in a different set of tasks can be reproduced and helped by Soft Computing and Computational Intelligence methodologies. Some traces of this research path can be found in [149–152]. A recent development of this research have discussed the role of transdisciplinarity and the relationship between cognitive science and cybernetics at a foundational level [121, 153], something which directly evokes the role of fuzziness in modern science. This has more recently lead to a discussion on how the same techniques can be fruitfully employed in the ongoing transformation of the idea of smart city (sensors everywhere) in cognitive city: colloquium everywhere, between humans and machines, machines and machines, in the spirit of Wiener [154, 155].

From my point of view (both physical and intellectual), the outlook for fuzziness in Italy is good. The variety of approaches that this contribution is testimony of has gifted Italy with a varied perspective, where fuzziness, more

that in other realities, has interacted and integrated with different disciplines in a coherent transdisciplinary fashion. During Gabriella Pasi's presidency of EuSFLaT, the Italian community of the association has grown both in numbers and participation. This has sublimated the lack of a strong national association (despite IEEE Fuzzy), and has improved the exchange between Italian scientists, as anyone who recently attended one of EuSFLaT conferences can testimony. I personally hope for the next fifty years for a complete dissolution of fuzziness, cognitive science, and all the other interested disciplines into a unique transdisciplinary science, beyond academical squabbles and disciplinary boundaries.

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